Basic Superposition with selection (1)

Basic Superposition right:

$$\frac{\Gamma' \to l \approx r, \Delta' \cdot \rho \quad \Gamma \to s \approx t, \Delta \cdot \rho}{\Gamma', \Gamma \to s[r]_p \approx t, \Delta, \Delta'\sigma \cdot \rho\sigma}$$

where:

• $s|_p$ is not a variable, $\sigma = mgu(s|_p\rho, l\sigma)$,

• $l\rho\sigma$ is strictly maximal among terms in $\Gamma'\rho\sigma \cup \{r\rho\sigma\}$ and $l\rho\sigma \approx r\rho\sigma$ is strictly maximal with respect to the equations in $\Delta'\rho\sigma$,

• nothing is selected in $\Gamma'\rho\sigma \to l\rho\sigma \approx r\rho\sigma, \Delta'\rho\sigma$

• $s\rho\sigma$ is strictly maximal among terms in $\Gamma\rho\sigma \cup \{t\rho\sigma\}$ and $s\rho\sigma \approx t\rho\sigma$ is strictly maximal with respect to the equations in $\Delta\sigma$

• nothing is selected in $\Gamma\rho\sigma \to s\rho\sigma \approx t\rho\sigma, \Delta\rho\sigma$

Basic Superposition with selection (2)

Basic Superposition left:

$$\frac{\Gamma' \rightarrow l \approx r, \Delta' \cdot \rho \quad \Gamma, s \approx t \rightarrow \Delta \cdot \rho}{\Gamma', \Gamma, s[r]_p \approx t \rightarrow \Delta, \Delta' \cdot \rho\sigma}$$

where:

- $s|_p$ is not a variable, $\sigma = mgu(s|_p\rho, l\rho)$,
- $l\rho\sigma$ is strictly maximal among terms in $\Gamma'\rho\sigma \cup \{r\rho\sigma\}$ and $l\rho\sigma \approx r\rho\sigma$ is strictly maximal with respect to the equations in $\Delta'\rho\sigma$,
- nothing is selected in $\Gamma'\rho\sigma \rightarrow l\rho\sigma \approx r\rho\sigma, \Delta'\rho\sigma$
- $t\rho\sigma \not\succeq s\rho\sigma$
- $s\rho\sigma \approx t\rho\sigma$ is selected in $\Gamma\rho\sigma, s\rho\sigma \approx t\rho\sigma \rightarrow \Delta\rho\sigma$ or nothing is selected in $\Gamma\rho\sigma, s\rho\sigma \approx t\rho\sigma \rightarrow \Delta\rho\sigma$ and $s\rho\sigma \approx t\rho\sigma$ is maximal with respect to the equations in $\Gamma\rho\sigma \cup \Delta\rho\sigma$

Basic Superposition with selection (3)

Basic Equality Resolution:

$$\frac{\Gamma, s \approx t \rightarrow \Delta \cdot \rho}{\Gamma \rightarrow \Delta \cdot \rho\sigma}$$

where:

• $\sigma = mgu(s\rho, t\rho)$,

• $s\rho\sigma \approx t\rho\sigma$ is selected in $\Gamma\rho\sigma, s\rho\sigma \approx t\rho\sigma \rightarrow \Delta\rho\sigma$ or nothing is selected in $\Gamma\rho\sigma, s\rho\sigma \approx t\rho\sigma \rightarrow \Delta\rho\sigma$ and $s\rho\sigma \approx t\rho\sigma$ is maximal in $\Gamma\rho\sigma \cup \Delta\rho\sigma$.

Basic Equality Factoring:

$$\frac{\Gamma \rightarrow s \approx t, s' \approx t', \Delta \cdot \rho}{\Gamma, t \approx t' \rightarrow s \approx t', \Delta \cdot \rho\sigma}$$

where:

• $\sigma = mgu(s\rho, s'\rho)$,

• $s\rho\sigma$ is strictly maximal among terms in $\Gamma\rho\sigma \cup \{t\rho\sigma\}$ and $s\rho\sigma \approx t\rho\sigma$ is maximal with respect to the equations in $\Delta\rho\sigma \cup \{s'\rho\sigma \approx t'\rho\sigma\}$,

• nothing is selected in $\Gamma\rho\sigma \rightarrow s\rho\sigma \approx t\rho\sigma, s'\rho\sigma \approx t'\rho\sigma, \Delta\rho\sigma$.