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1. Exercises for the Course „Automata and Logic“

Exercise 1:

Consider the algorithm for graph reachability presented in the lecture. Prove that the algorithm returns “yes” if and only if the node v is reachable from the node u.

Note: since it follows from the complexity analysis given in the lecture that the algorithm terminates, the latter can be assumed without proof.

Exercise 2:

Consider the algorithm for graph reachability presented in the lecture. Assume that the graph $G = (V, E)$ is given in the following form:

- V consists of the numbers $1, \ldots, n$;
- E is given as an array A of length n, where the value of $A(i)$ is $\{j \mid (i, j) \in E\}$.

With this data structure, can you give a better runtime bound than n^2 (in terms of $|V|$ and/or $|E|$) for the same algorithm? In which sense is the bound better?

Exercise 3:

Determine which of the following statements are correct. Explain your decision.

(a) $2n = O(n)$, $2n = o(n)$, $1 = o(n)$;

(b) $n = O(n \cdot \log n)$, $n \cdot \log n = o(n^2)$;

(c) $2^{2^n} = O(2^n)$, $\sqrt{n} = o(n)$;

(d) $f(n) = o(g(n))$ implies $f(n) = O(g(n))$ and $g(n) \neq O(f(n))$;

(e) Consider the following functions $f(n)$ and $g(n)$, and determine whether $f(n) = O(g(n))$, $g(n) = O(f(n))$, and $f(n) = o(g(n))$:

- $f(n) = \begin{cases} 2^n & \text{if } n \text{ even} \\ n & \text{if } n \text{ odd} \end{cases}$
- $g(n) = 0.5 \cdot n$;

Exercise 4:

It was sketched in the lecture how to decide the satisfiability of Horn formulas in linear time by introducing a counter for each implication.

Work out the details of this idea by writing a linear-time algorithm for the satisfiability of Horn formulas. Write the algorithm in pseudo-code notation (as in the lecture).

Exercise 5:

Prove that the graph reachability algorithm is a special case of the algorithm for deciding satisfiability of Horn formulas. Hint: introduce one variable for each node of the graph.