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2. Exercises for the Course „Automata and Logic“

Exercise 6:

Show how to enumerate the nodes of a tree such that each node is enumerated before its parent. Make sure (and prove) that your enumeration can be computed in time polynomial in the number of nodes of the tree.

Exercise 7:

(a) Find two graphs G_1 and G_2 with nodes $u \in V_1$ and $v \in V_2$ such that there exists a simulation $\stackrel{\rightharpoonup}{\sim}$ with $u \stackrel{\rightharpoonup}{\sim} v$, but no homomorphism h with $h(u) = v$.

(b) Let G_1 be a tree and G_2 a graph, and let $u \in V_1$ and $v \in V_2$. Prove or refute the following:

- if there is a homomorphism with $h(u) = v$, then there exists a simulation $\stackrel{\rightharpoonup}{\sim}$ with $u \stackrel{\rightharpoonup}{\sim} v$.
- if there is a simulation $\stackrel{\rightharpoonup}{\sim}$ with $u \stackrel{\rightharpoonup}{\sim} v$, then there exists a homomorphism with $h(u) = v$.

(c) Let G_1 be a tree and G_2 a graph. Compare the algorithm for computing tree homomorphisms applied to G_1 and G_2 with the algorithm for computing simulations applied to G_1 and G_2.

(d) Can the polynomial-time algorithm for computing tree-homomorphisms be adapted to the case where G_1 is a graph and G_2 is a tree?

Exercise 8:

(a) Prove that, for all graphs G_1 and G_2, the class of simulations from G_1 to G_2 is closed under union: if $\stackrel{\rightharpoonup}{\sim}_1$ and $\stackrel{\rightharpoonup}{\sim}_2$ are simulations from G_1 to G_2, then $\stackrel{\rightharpoonup}{\sim}_1 \cup \stackrel{\rightharpoonup}{\sim}_2$ is also a simulation from G_1 to G_2.

(b) Use (a) to prove that, for all graphs G_1 and G_2, there exists a unique greatest simulation from G_1 to G_2.

(c) Let G_1 and G_2 be graphs, and let $u \in V_1$ and $v \in V_2$. Prove or refute the following:

- The class of simulations from G_1 to G_2 is closed under intersection.
- There exists a simulation from G_1 to G_2.
- There exists a simulation $\stackrel{\rightharpoonup}{\sim}$ from G_1 to G_2 with $u \stackrel{\rightharpoonup}{\sim} v$.

Exercise 9:

Convert the following pairs C_1, C_2 of $\mathcal{EL}$ concepts into trees. Check the existence of homomorphisms between the trees to decide whether $C_1 \sqsubseteq C_2$.

C_1	C_2
$\exists r.(A \sqcap B)$	$\exists r.A \sqcap \exists r.B$
$\exists r.\top \sqcap \exists r.\top$	$\top \sqcap \exists r.\top$
$\exists r.\exists r.A \sqcap \exists r.\exists s.B$	$\exists r.(\exists r.A \sqcap \exists s.B)$
$\exists r.(A \sqcap B \sqcap \exists r.A \sqcap \exists r.B)$	$\exists r.(A \sqcap \exists r.A \sqcap \exists r.B) \exists r.(B \sqcap \exists r.A \sqcap \exists r.B)$

Exercise 10:

Let G_1 and G_2 be graphs. A *bisimulation* is a relation $\sim \subseteq V_1 \times V_2$ satisfying the following properties:

- $u \sim v$ implies $\ell_1(u) = \ell_2(v)$;
- $(u_1, r, u_2) \in E_1$ and $u_1 \sim v_1$ implies that there is a $v_2 \in V_2$ with $(v_1, r, v_2) \in E_2$ and $u_2 \sim v_2$;
- $(v_1, r, v_2) \in E_2$ and $u_1 \sim v_1$ implies that there is a $u_2 \in V_1$ with $(u_1, r, u_2) \in E_1$ and $u_2 \sim v_2$.

Do the following:

(a) Develop an algorithm that, given two graphs G_1, G_2 and nodes $u \in V_1$ and $v \in V_2$, determines the existence of a bisimulation $\sim$ with $u \sim v$. Make sure the algorithm runs in polynomial time.

(b) Prove or refute the following: there exists a bisimulation between two graphs G_1 and G_2 with $u \sim v$ iff there exists a simulation $\vec{\sim}$ (where the comparison operator λ is equality) from G_1 to G_2 with $u \,\vec{\sim}\, v$, and a simulation $\vec{\sim}$ from G_2 to G_1 with $v \,\vec{\sim}\, u$.