Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

4. Exercises for the Course „Automata and Logic“

Note: In all exercises asking for a Turing machine, describe the machine in a similar way as the machine for graph reachability was described in the lecture. Do not give the detailed transition tables.

Exercise 14:

We want to construct a Turing machine that implements the algorithm for graph reachability given in the lecture. If the input graph consists of the nodes $\{1, \ldots, n\}$, then the input to the Turing machine is given as a word

$$m_1 \cdots m_{n^2} \# u_1 \cdots u_k \# v_1 \cdots v_k$$

where

- $m_1 \ldots m_{n^2} \in \{0,1\}^*$ is obtained by concatenating all rows of the adjacency matrix of the input graph (as described in the lecture);
- $u_1 \cdots u_k \in \{0,1\}^*$ is a binary representation of the start node u;
- $u_1 \cdots u_k \in \{0,1\}^*$ is a binary representation of the destination node v;

Assume that the algorithm implements the graph reachability algorithm by marking edges (instead of nodes): it simply replaces entries $0, 1$ of the adjacency matrix with marked variants $0', 1'$. Describe how the Turing machine can solve the following subtasks:

(a) determine the number of nodes in the input graph;

(b) given a node i in binary representation, find a node j that is a direct successor of i such that the edge from i to j is unmarked (if such a node exists).

Describe how the Turing machine works using these subtasks as blackbox modules.

Exercise 15:

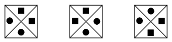
Let Σ be a finite alphabet. Describe a Turing machine that enumerates all (finite) words over Σ such that shorter words are enumerated before longer ones. Can you also enumerate all (finite) words over $\mathbb{N}$? How?

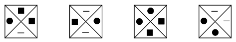
Exercise 16:

Give a proof of Corollary 2.3 from the lecture: show that every language that is decidable by an NTM is also decidable by a DTM. Hint: Use the Turing machine from Exercise 15, find an appropriate adaptation of the construction from the proof of Theorem 2.2, and use König's lemma.

Exercise 17:

Check whether or not the following domino systems have a solution:

(a) 

(b) 

Exercise 18:

The domino systems for the lecture require the tiling of the first quadrant of the plane: solutions τ have domain $\mathbb{N} \times \mathbb{N}$. By changing the domain of τ, we obtain versions of the domino problem that require tilings of the full plane $\mathbb{Z} \times \mathbb{Z}$ or of an n-square $\{0, \ldots, n-1\} \times \{0, \ldots, n-1\}$. Prove the following:

- if a domino system $\mathcal{D} = (D, H, V)$ can tile the n-square, for each $n \geq 0$, then $\mathcal{D}$ can tile the first quadrant of the plane. Hint: Use König's lemma.
- if a domino system $\mathcal{D} = (D, H, V)$ can tile the first quadrant of the plane, then it can tile the whole plane.