Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

5. Exercises for the Course „Complexity and Logic“

Exercise 19:

The description logic $\mathcal{ALCFI}$ extends $\mathcal{ALCF}$ with inverse roles that can be used inside existential and universal restrictions: concept terms of $\mathcal{ALCFI}$ are formed as concept terms of $\mathcal{ALCF}$, but may additionally use the constructors $\exists f^-.C$ and $\forall f^-.C$. The semantics of the new constructors is as follows:

- $(\exists f^-.C)^\mathcal{I} = \{d \in \Delta^\mathcal{I} \mid \exists e \in \Delta^\mathcal{I} : f^\mathcal{I}(e) = d \ \wedge \ e \in C^\mathcal{I}\}$;
- $(\forall f^-.C)^\mathcal{I} = \{d \in \Delta^\mathcal{I} \mid \forall e \in \Delta^\mathcal{I} : f^\mathcal{I}(e) = d \ \rightarrow \ e \in C^\mathcal{I}\}$;

An $\mathcal{ALCFI}$ concept C is *satisfiable* iff there exists an interpretation $\mathcal{I}$ such that $C^\mathcal{I} \neq \emptyset$ (Note: there is no reference to TBoxes as in the case of $\mathcal{ALCF}$).

Use a reduction of the domino problem to prove that satisfiability of $\mathcal{ALCFI}$ concepts is undecidable.

Hint: while features are interpreted as functional relations, the inverses of features are not necessarily functional. Proceed as in the $\mathcal{ALCF}$ case and simulate TBoxes by ensuring that (i) models of the reduction concept have a root, and (ii) from this root, every other element of the model can be reached via the inverse of some feature g.

Exercise 20:

Let L be a language. Then the *closure under Kleene Star of* L is defined as

$$L^* := \{w_1 \cdots w_k \mid k \geq 0 \wedge w_1, \ldots, w_k \in L\}.$$

Prove the following:

(a) If L, L' are in P, then $L \cup L'$ is in P.

(b) If L is in P, then L^* is in P.

Hint: For (b), it is convenient to use dynamic programming techniques.

Exercise 21:

A language L is *in* LogSpace if it is decided by a 2-tape Turing machine such that

- the first tape is read-only and contains the input and
- only $d \cdot \mathsf{log}(n)$ cells are used on the second tape on inputs of length n, for some $d \in \mathbb{N}$.

Prove that the language $L = \{a^n b^n \mid n \geq 0\}$ is in LogSpace.

Exercise 22:

Prove the space compression theorem (Theorem 3.8 from the lecture): for all $\varepsilon \in \mathbb{R}^+$ and all $S : \mathbb{N} \rightarrow \mathbb{R}^+$, we have $\mathsf{DSpace}(S) \subseteq \mathsf{DSpace}(\mathsf{max}(n, \varepsilon \cdot S(n)))$.