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6. Exercises for the Course „Complexity and Logic“

Exercise 23:

Time and space are only two examples of complexity measures for computations. The following general approach is known as *Blum complexity*. Let f be a function mapping pairs (M, w), with M a (deterministic) Turing machine and w an input for M, to nonnegative integers. Then f is a *complexity measure* if it satisfies the following conditions:

(i) $f(M, w)$ is defined if and only if M terminates on w;

(ii) Given M, w, k, it is decidable whether $f(M, w) = k$.

Note that there may be M, w, k given as input such that M does not halt on w. Consider the following functions:

- $\mathit{space}'_M(w)$ is defined as the function $\mathit{space}_M(w)$ in the lecture, but it is undefined if M does not terminate on w.
- $\mathit{ink}_M(w)$ is the number of times during the finite computation of M on w that a symbol has to be overwritten by a different symbol. It is undefined if M does not terminate on w.
- $\mathit{carbon}_M(w)$ is the number of times during the finite computation of M on w that a symbol has to be overwritten by the same symbol. It is undefined if M does not terminate on w.

Show the following:

(a) $\mathit{time}_M(w)$ and $\mathit{space}'_M(w)$ are complexity measures.

(b) $\mathit{ink}_M(w)$ is a complexity measure.
Hint: the proof of (ii) involves show the following: if $\mathit{ink}_M(w) = k$, then the number of different configurations that the computation of M on w may take is finite.

(c) $\mathit{carbon}_M(w)$ is not a complexity measure.
Hint: To prove that (ii) is violated, reduce the word problem to the problem "$\mathit{carbon}(M, x) = k$?".

Exercise 24:

Complete the proof of Lemma 3.10 from the lecture: let S, M, and X be as in the proof sketch given in the lecture. Show that, for words $w \in \Sigma^* \setminus X$, the question "$w \in L(M)$?" can be decided by simulating M using only space $S(|w|)$.

Exercise 25:

Complete the proof of Lemma 3.11 from the lecture: show that the function t described in the proof sketch given in the lecture is computable.

Exercise 26:

Prove Theorem 3.12 from the lecture, i.e., the gap theorem for time: for every total computable function g with $g(n) \geq n$, there is a total computable function T with $DTime(T) = DTime(g \circ T)$.