Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

8. Exercises for the Course „Complexity and Logic“

Exercise 31:

Show that the following problem is in PSpace: given a non-deterministic finite automaton (NFA) $\mathcal{A}$ over an alphabet Σ, is $L(\mathcal{A}) = \Sigma^*$?

Hint: prove that, if $L(\mathcal{A}) \neq \Sigma^*$ and $\mathcal{A}$ has n states, then there exists a word $w \in \Sigma^*$ of length at most 2^n such that $w \notin L(\mathcal{A})$. Then use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch's theorem.

Exercise 32:

Relate the following classes of languages to the classes P, NP, and PSpace (by means of set inclusion):

(a) the class of regular languages;

(b) the class of context-free languages;

(c) the class of context-sensitive languages.

Exercise 33:

Prove that the following problems can be solved in polynomial time:

- given a Boolean formula φ using variables $p_1, \dots, p_k$ and a truth assignment $w \in \{0,1\}^k$, check whether w is a model of φ.
- given an $\mathcal{ALCF}$ concept C (as introduced in the lecture) and a finite interpretation $\mathcal{I}$, check whether $C^{\mathcal{I}} \neq \emptyset$.

Exercise 34:

An *acyclic* finite automaton (AFA) is a non-deterministic finite automaton whose transition relation is acyclic, i.e., there is no sequence $a_0, \dots, a_{n-1}$ of symbols and $q_0, \dots, q_n$ of states such that $q_0 = q_n$ and

$$q_{i+1} \in \delta(q_i, a_i) \text{ for all } i < n.$$

(In other words, AFAs are exactly the non-deterministic finite automata that accept finite languages). Prove that the following problem is in NP: given AFAs $\mathcal{A}_1$ and $\mathcal{A}_2$, does $L(\mathcal{A}_1) \not\subseteq L(\mathcal{A}_2)$ hold?

Exercise 35:

The description logic $\mathcal{EL}$ has been introduced in the lecture. The extension $\mathcal{ELU}$ of $\mathcal{EL}$ is obtained by adding disjunction "$\sqcup$" as a concept-forming constructor. Subsumption between $\mathcal{ELU}$ concept terms ("$C \sqsubseteq D$") is defined as for $\mathcal{EL}$. Prove that the following problem is in NP: given $\mathcal{ELU}$ concept terms C and D, does $C \not\sqsubseteq D$ hold?

Hint: prove the following: if there exists an interpretation $\mathcal{I}$ with $D^{\mathcal{I}} \setminus C^{\mathcal{I}} \neq \emptyset$, then there is such an $\mathcal{I}$ with $|\Delta^{\mathcal{I}}|$ bounded by the length of D. Proceed in two steps: first, show that if $x \in D^{\mathcal{I}}$, then $x \in E^{\mathcal{I}}$ for some $\mathcal{EL}$-concept E such that $E \sqsubseteq D$ and E is not longer than D. Second, show that any model of $E^{\mathcal{I}}$ can be converted into a model of $E^{\mathcal{I}}$ whose cardinality is bounded by the number of nodes in the $\mathcal{EL}$-tree of E.