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10. Exercises for the Course „Complexity and Logic“

Exercise 41:

Complete the 3rd step in the proof of Theorem 4.11: consider the formulas

$$\begin{aligned}\psi_1 &:= (\neg p_1 \vee p_3) \wedge \neg p_2\\ \psi_2 &:= y_0 \wedge (y_0 \leftrightarrow (y_1 \wedge \neg p_2)) \wedge (y_1 \leftrightarrow (\neg p_1 \vee p_3)).\end{aligned}$$

Show that ψ_1 is satisfiable iff ψ_2 is satisfiable. Also show that these two formulas are not equivalent.

Exercise 42:

Complete the proof of Theorem 4.17 from the lecture: show that a mapping $h : V_1 \to V_2$ is a homomorphism from G_1 to G_2 if and only if h is a homomorphism from $\widehat{G}_1$ to $\widehat{G}_2$.

Exercise 43:

Complete the proof of Theorem 4.19 from the lecture: show that the containment of conjunctive queries can be polynomially reduced to conjunctive query answering.

Exercise 44:

Complete the proof of Theorem 4.20 from the lecture: show that there exists a homomorphism $h : G_1 \to G_2$ with $h(u_0) = v_0$ iff x_1 is an answer for K w.r.t. $\mathcal{I}$.

Exercise 45:

A conjunctive query K is called *tree-shaped* if it satisfies the following conditions:

(a) K has exactly one answer variable a;

(b) the maximum arity of predicates occurring in K is two;

(c) the graph $G_K = (V_K, E_K)$ is tree with root a, where

- V_K is the set of variables occurring in K;
- $(x, y) \in E_K$ iff $P(x, y)$ is an atom in K for some relation P.

Show that the query containment problem $K_1 \leq K_2$ can be solved in polynomial time if K_1 is tree-shaped. Is the problem still in polynomial time if we drop requirement (a) and relax (c) by demanding that G_K is a collection of disjoint trees whose roots are exactly the answer variables?