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11. Exercises for the Course „Complexity and Logic“

Exercise 46:

Show that, if P = NP, then there exists a polynomial time algorithm that, given a formula φ of propositional logic, produces a satisfying assignment for φ (if it is satisfiable).

Exercise 47:

Show that every PSPACE-hard problem is also NP-hard.

Exercise 48:

Which of the following QBFs is valid?

(a) $\forall p_1 \exists p_2 \forall p_3 (p_2 \rightarrow (p_1 \vee p_3))$

(b) $\forall p_1 \exists p_2 \forall p_3 ((p_1 \vee p_2) \rightarrow p_3)$

(c) $\forall p_1 \forall p_2 \forall p_3 \exists q_1 \exists q_2 (q_1 \leftrightarrow ((p_1 \vee p_2) \wedge (\neg p_1 \vee \neg p_2)) \wedge (q_2 \leftrightarrow ((q_1 \vee p_3) \wedge (\neg q_1 \vee \neg p_3)))$

Exercise 49:

Let $\psi = Q_1 p_1 \dots Q_n p_n . \varphi$ be a QBF. A *witness tree* for ψ is a labelled tree $T = (V, E, \ell)$ such that

- $\ell : V \rightarrow \{0, 1\}$ is a node labelling function;
- if $v \in V$ is on level $i < n$ of T (the root is on level 0), then v has 1 successor if $Q_{i+1} = \exists$ and 2 successors if $Q_{i+1} = \forall$;
- if $v \in V$ is on level n of T and the path from the root to v is $v_0, \dots, v_n$, then the following mapping is a truth assignment that makes φ true:

$$\tau(p_i) := \ell(v_i) \text{ for } 1 \leq i \leq n$$

Prove that a QBF ψ is valid iff there exists a witness tree T for ψ.