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13. Exercises for the Course „Complexity and Logic“

Exercise 55:

Prove Lemma 5.11 from the lecture:

- The QBF ψ is valid iff C_ψ is satisfiable;
- C_ψ can be constructed in polynomial time from ψ.

Exercise 56:

Let $\mathcal{ALCQ}$ be the extension of $\mathcal{ALC}$ with the concept constructor $(\geqslant n\ r\ C)$, where n is a non-negative integer, r is a role name, and C a concept. The semantics of the new constructor is given as follows:

$$(\geqslant n\ r\ C)^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid \{e \in \Delta^{\mathcal{I}} \mid (d,e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\} \text{ has cardinality at least } n\}.$$

Modify the $\mathcal{ALC}$-World algorithm from the lecture such that it can decide satisfiability of $\mathcal{ALCQ}$-concepts.

Exercise 57:

Formulas of $FO^=$, the first-order logic of equality, are inductively defined as follows:

- If u and v are variables, then $(u = v)$ is a formula;
- if φ and ψ are formulas and u is a variable, then the following are also formulas $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\exists u.\varphi$, and $\forall u.\varphi$.

A $FO^=$ formula is a *sentence* if it does not have free variables. The semantics is defined as usual in first-order logic with equality. Note that models consist only of a universe (of arbitrary, but non-zero cardinality) since there are no function symbols and no uninterpreted predicate symbols in the syntax.

The first-order theory of equality is the set of all $FO^=$ sentences that are valid. Show by a QBF reduction that the first-order theory of equality is PSpace-hard.