Exercise Sheet 1

for the lecture

Complexity Theory

Exercise 1

Words over the alphabet $\Sigma = \{0,1\}$ can be viewed as the binary representation of a non-negative integer (lowest bit rightmost, possibly with additional leading zeros). Construct a 1-tape DTM that works over the alphabet Σ and does the following: when started on an input word w representing the number $n \in \mathbb{N}$, it converts w into the binary representation of $n+1$ and stops in an accepting state. To describe the transition function, draw the state diagram rather than using tuple notation.

Exercise 2

A *palindrome* is a word that reads the same backwards, e.g. "reittier" or "malayalam". Find a k-tape DTM (for some self-chosen k) that works over the alphabet $\Sigma = \{a, b\}$ and decides whether its input is a palindrome. The first tape is the input tape (and thus cannot be modified). On every other tape, at most $\mathcal{O}(\log(n))$ cells may be used. It suffices to describe the machine on an intuitive level.

Exercise 3

Let a 1^*-DTM be a DTM that is equipped with a single *two-side* infinite tape. Show that every language decided by a 1^*-DTM is also decided by a 1-DTM within the same time and space bounds (up to a constant).

Exercise 4

For f, g function from $\mathbb{N}$ to $\mathbb{N}$, we write $f \in \Theta(g)$ if $f \in \mathcal{O}(g)$ and $g \in \mathcal{O}(f)$. Intuitively, $f \in \Theta(g)$ if f and g have the same rate of growth. Show the following: The rate of growth of a polynomial is captured by the polynomial's first non-zero term, i.e.: if $p(n)$ is a polynomial of degree d, then $p(n) \in \Theta(n^d)$.

Exercise 5

Complete Theorem 3 by showing that $(d^{T(n)+1} - 1) \cdot c \cdot T(n) \in 2^{\mathcal{O}(T(n))}$.