Exercise Sheet 2

for the lecture

Complexity Theory

Exercise 1

Prove Lemma 24: All $\mathcal{C} \in \{\mathrm{P}, \mathrm{NP}, \mathrm{PSpace}, \mathrm{ExpTime}, \mathrm{NExpTime}, \mathrm{ExpSpace}\}$ are closed under polytime reductions, i.e., if $L' \in \mathcal{C}$ and $L \leq_p L'$, then $L \in \mathcal{C}$

Exercise 2

Finish the proof of Theorem 27 by showing that ψ_w is satisfiable iff M accepts w.

Exercise 3

Prove Lemma 28: If $\mathcal{C}$ is a complexity class closed under polytime reductions, then L being $\mathcal{C}$-hard and $L \leq_p L'$ implies that L' is $\mathcal{C}$-hard.

Exercise 4

HALF-CLIQUE is the problem to decide, given a graph G with $n \geq 1$ nodes, whether G contains a clique of size $n/2$. Prove that HALF-CLIQUE is NP-complete (Hint: to show NP-hardness, use a reduction from CLIQUE).

Exercise 5

SET-SPLITTING is the following problem: given a set S and a collection $C = S_1, \ldots, S_k$ of subsets of S, for some $k > 0$, decide whether all elements of C can be colored *red* or *black* such that no S_i has all its elements colored with the same color. Show that SET-SPLITTING is NP-complete (Hint: to show NP-hardness, use a reduction from 3SAT).