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10. Exercises for the lecture **Term Rewriting Systems**

Exercise 41:

A TRS R is called *right-irreducible* if each r with $\ell \to r \in R$ is irreducible.
Prove or disprove the following claim: if R is right-ground and right-irreducible, then R is terminating.

Exercise 42:

Prove Lemma 5.20 of the lecture:

Lemma 5.20 Let $\mathcal{A}$ be a monotone polynomial interpretation of Σ. Then $f^{\mathcal{A}}$ is a montone function for each $f \in \Sigma$.

Exercise 43:

Let R be a finite term rewriting system. We use $|t|_f$ to denote the number of occurrences of the symbol $f \in \Sigma$ in a term t. Prove the following claim:

There is a natural number k_R such that, for all terms s, t, if $s \xrightarrow[R]{} t$, then $|t|_f \leq k_R(|s|_f + 1)$.

Exercise 44:

Let R_{Ack} be the (terminating) TRS from the lecture with

$$R_{Ack} = \{a(0, y) \longrightarrow s(y),\ a(s(x), 0) \longrightarrow a(x, s(0)),\ a(s(x), s(y)) \longrightarrow a(x, a(s(x), y))\}.$$

Prove that there is no primitiv-recursive function that provides an upper bound for the length of reductions, given the length of the first term. More precisely, prove that there is no primitiv-recursive function f such that, if $t_1 \xrightarrow[R_{Ack}]{} t_2 \xrightarrow[R_{Ack}]{} \dots \xrightarrow[R_{Ack}]{} t_k$, then $k \leq f(|t_1|)$.

Hint: Use Exercise 43 and the fact that the Ackermann function grows faster than any primitiv-recursive function.