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13. Exercises for the lecture **Term Rewriting Systems**

Exercise 54:

Complete the proof of Theorem 6.1 of the lecture: let E be a set of identities over Σ. Prove the following equivalence for all terms $s, t \in \mathbf{T}(\Sigma, V)$, where $\mathsf{const}(\cdot)$ is a function that replaces every occurrence of a variable in x with a constant $a_x \notin \Sigma$:

$$s \approx_E t \text{ iff } \mathsf{const}(s) \approx_E \mathsf{const}(t)$$

Exercise 55:

Find terms r_1, r_2 such that $\{f(g(x)) \longrightarrow r_1,\ g(h(x)) \longrightarrow r_2\}$ is confluent.

Exercise 56:

Recall Definition 5.14: for a non-empty Σ-algebra $\mathcal{A}$ and $>$ a well-founded order on the carrier set A of $\mathcal{A}$, the reduction order $>_{\mathcal{A}} \subseteq \mathcal{T}(\Sigma, V) \times \mathcal{T}(\Sigma, V)$ induced by $\mathcal{A}$ is defined as follows:

$$s >_{\mathcal{A}} t \text{ iff } \varphi(s) > \varphi(t) \text{ for all homomorphisms } \varphi : \mathcal{T}(\Sigma, V) \to A.$$

Now let $>_{\mathcal{A}}, >_{\mathcal{B}}$ be reduction orders induced by $\mathcal{A}$ and $\mathcal{B}$. We define the relation $>_{\mathcal{A}\times\mathcal{B}}$ as follows: for $s, t \in \mathcal{T}(\Sigma, V)$, we define

$$\begin{array}{ll} s >_{\mathcal{A}\times\mathcal{B}} t \text{ iff} & s >_{\mathcal{A}} t \text{ or} \\ & \text{for all homomorphisms } \varphi : \mathcal{T}(\Sigma, V) \to A, \text{ we have } \varphi(s) = \varphi(t) \text{ and } s >_{\mathcal{B}} t. \end{array}$$

(a) Prove that, if $>_{\mathcal{A}}$, $>_{\mathcal{B}}$ are reduction orders induced by $\mathcal{A}$ and $\mathcal{B}$, then $>_{\mathcal{A}\times\mathcal{B}}$ is a reduction order.

(b) Use this fact to prove termination of the following TRS:

$$\begin{array}{rcl} \{s(x+y) & \longrightarrow & x + s(y)\} \cup \\ \{x + s^n(0) & \longrightarrow & s^n(x) \mid \text{ for all } n \geq 0\} \end{array}$$

Exercise 57:

Finish Example 6.8 from the lecture: show that the TRS $\{f(f(x)) \longrightarrow g(x),\ f(g(x)) \longrightarrow g(f(x))\}$ is terminating and confluent.

Exercise 58:

Consider the decision procedure in Corollary 6.7 for the confluence of finite, terminating TRSs. Try to (!) establish an upper bound for the run time of the procedure as a function of the size of the input TRS.