Lehrstuhl für Automatentheorie
Technische Universität Dresden
Prof. Dr. F. Baader

01187 Dresden
Tel.: 0351/463–38314

16. Exercises for the lecture **Term Rewriting Systems**

Exercise 65:

The ideal $< f_1, \dots, f_n >$ generated by polynomials $f_1, \dots, f_n \in K[X_1, \dots, X_n]$ is defined as

$$< f_1, \dots, f_n >:= \{g_1 f_1 + \dots + g_k f_k \quad | \quad g_1, \dots, g_k \in K[X_1, \dots, X_n]\}$$

Prove that $< f_1, \dots, f_n >$ is indeed an ideal.

Exercise 66:

Given a finitely generated ideal of polynomials, $J =< f_1, \dots, f_k >$, where $f_1, \dots, f_2 \in K[X_1, \dots, X_n]$, we define

$$f \equiv_J \text{ iff } f - g \in J.$$

Prove that $\equiv_J$ is a congruence of the polynomial ring $K[X_1, \dots, X_n]$.

Exercise 67:

We define an order on monomials $>$ in the following way:

$X_1^{k_1} \cdots X_n^{k_n} > X_1^{l_1} \cdots X_n^{l_n}$ iff

(a) $\Sigma_{i=1}^n k_i > \Sigma_{i=1}^n l_i$ or

(b) $\Sigma_{i=1}^n k_i = \Sigma_{i=1}^n l_i$ and $(k_1, \dots, k_n) >_{lex}^n (l_1, \dots, l_n)$.

Show that $>$ is admissible.