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5. Exercises for the lecture **Term Rewriting Systems**

Exercise 20:

The reduction relation $\longrightarrow$ enjoys the *diamond property* if

for all x, y_1, y_2: if $y_1 \longleftarrow x \longrightarrow y_2$, then there is some z with $y_1 \longrightarrow z \longleftarrow y_2$

Prove that, if $\longrightarrow$ enjoys the diamond property, then every element x is either in normal form or does not have a normal form.

Exercise 21:

Consider the terms $s = f(x, g(h(k(k(y)), x), z), h(x, y))$ and $t = g(z, h(x, k(k(y))))$. Describe $t|_1$, $t|_{1111}$, $t|_{11111}$, $t[s]_2$, and $t[s|_2]_{21}$.

Exercise 22:

Prove Part 2 of Lemma 3.4 by induction on the length of words denoting positions: if $p \in \mathsf{pos}(s)$ and $q \in \mathsf{pos}(t)$, then

$$\begin{aligned} (s[t]_p)|_{pq} &= t|_q \\ (s[t]_p)[r]_{pq} &= s[t[r]_q]_p \end{aligned}$$