Lehrstuhl für Automatentheorie
Technische Universität Dresden
Prof. Dr. F. Baader

01187 Dresden
Tel.: 0351/463–38314

8. Exercises for the lecture **Term Rewriting Systems**

Exercise 33:

Prove the undecidability of the *uniform* Halting problem (given a Turing machine $\mathcal{M}$, does $\mathcal{M}$ always terminate when started on k, for all inputs k?) by a reduction of the Halting problem (given a Turing machine $\mathcal{M}$ and an input k, does $\mathcal{M}$ always terminate when started on k?).

Exercise 34:

Why do we need $\vec{\cdot}$ in the proof of Theorem 5.7? More precisely, for a Turing machine $\mathcal{M}$, let

$$\Sigma'_{\mathcal{M}} := \{s_0, \ldots, s_n\} \cup \{q_0, \ldots, q_p\} \cup \{\overrightarrow{\ell}, \overleftarrow{r}\}$$

and let $R'_{\mathcal{M}}$ be the rewrite system obtained from $R_{\mathcal{M}}$ by replacing both $\overrightarrow{s_i}$ and $\overleftarrow{s_i}$ with s_i. Give an example of a terminating Turing machine $\mathcal{M}$ for which $R'_{\mathcal{M}}$ does not terminate.

Exercise 35:

Prove that the following is *not* a decision procedure for the termination of a ground term rewrite system $R = \{\ell_i \to r_i \mid 1 \leq i \leq n\}$:
Generate all reduction sequences starting with r_1. If one of these sequences yields a term that has r_1 as a subterm, then answer "non-terminating". Otherwise, continue with r_2, etc.

Exercise 36:

Prove the "left version" of Theorem 5.9: Termination of a finite, left-ground term rewrite system is decidable.

Exercise 37:

We define the order $>$ on terms as follows (recall that $|s|$ denotes the size of s, i.e., the number of positions in s, and $|s|_x$ the number of occurrences of x in s):

$$s > t \text{ if } |s| > |t| \text{ and } |s|_x \geq |t|_x, \text{ for all } x \in V.$$

Prove that $>$ is a reduction order on $\mathcal{T}(\Sigma, V)$.