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9. Exercises for the lecture **Term Rewriting Systems**

Exercise 38:

The following problem is known as *Hilbert's 10th problem*, and it has been proven to be undecidable:
Given: Two polynomials $P, Q \in \mathbb{N}[X_1, \ldots, X_n]$.
Question: Are there $a_1, \ldots, a_n \in \mathbb{N}$ such that $P(a_1, \ldots, a_n) = Q(a_1, \ldots, a_n)$?

Show that the undecidability of Hilbert's 10th problem implies the undecidability of the following problem (which you know from the lecture):
Given: Two polynomials $P, Q \in \mathbb{N}[X_1, \ldots, X_n]$ and a decidable set $A \subseteq \mathbb{N} \setminus \{0\}$.
Question: Is $P >_A Q$? That is, is $P(a_1, \ldots, a_n) > Q(a_1, \ldots, a_n)$ for all $a_1, \ldots, a_n \in A$?

Exercise 39:

(a) Let $P \in \mathbb{Z}[X]$ be a polynomial in one indeterminate and coefficients in $\mathbb{Z}$, i.e. $P(X) = a_n X^n + a_{n-1} X^{n-1} + \cdots + a_0$, for $a_0, \ldots, a_n \in \mathbb{Z}$.

Prove for all $r \in \mathbb{Z}$: if $P(r) = 0$, then $r | a_0$, i.e. any root of P divides a_0.

(b) Devise a decision procedure which for each polynomial $P \in \mathbb{Z}[X]$ decides whether P has a root in $\mathbb{Z}$ (i.e. if there is an $r \in \mathbb{Z}$, such that $P(r) = 0$) .

(c) Show that in polynomials with more than 1 indeterminate, the roots need not satisfy such property.

Exercise 40:

Consider the TRS $R = \{g(x, g(y, z)) \longrightarrow g(g(x, y), z), \quad g(g(x, y), z) \longrightarrow g(y, y)\}$. Use a polynomial interpretation to prove that R terminates. Hint: try $p_g = xy + y^2$.