Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nthnitzer Str. 46,
01187 Dresden
Tel.: 0351/463–38237
Tel.: 0351/463–39171

10. Exercises for the Course „Automata and Logic“

Exercise 42:

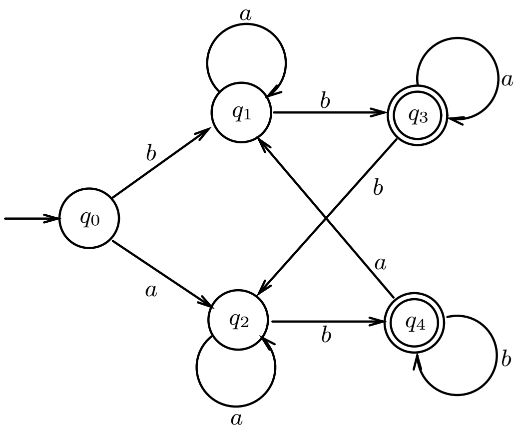
(a) Show that the construction used in the proof of Lemma 4.7.1 does not work for automata whose initial state is reachable from another state by giving a counter example.

(b) Complete the proof of Lemma 4.7 from the lecture by showing the following:

If $L_1, L_2 \subseteq \Sigma^\omega$ are Büchi–recognizable, then $L_1 \cup L_2$ is also Büchi–recognizable.

Exercise 43:

Let $\Sigma := \{a, b\}$ and $L \subseteq \Sigma^\omega$ the language recognized by the following Büchi–automaton:



Determine an *ω–regular expression* for L, i.e. find a number $n \in \mathbb{N} \setminus \{0\}$ and regular languages $U_1, V_1, \ldots, U_n, V_n \subseteq \Sigma^*$ such that

$$\bigcup_{i=1}^{n} U_i \cdot V_i^\omega = L\,.$$

Exercise 44:

Let Σ be an alphabet and $L, L_1, L_2 \subseteq \Sigma^*$. Prove or refute:

(a)
- $(L_1 \cup L_2)^\omega \subseteq L_1^\omega \cup L_2^\omega$
- $(L_1 \cup L_2)^\omega \supseteq L_1^\omega \cup L_2^\omega$

(b)
- $\lim(L_1 \cup L_2) \subseteq \lim L_1 \;\cup\; \lim L_2$
- $\lim(L_1 \cup L_2) \supseteq \lim L_1 \;\cup\; \lim L_2$

(c)
- $L^\omega \subseteq \lim L^+$
- $L^\omega \supseteq \lim L^+$

(d)
- $\lim(L_1 \cdot L_2) \subseteq L_1 \cdot L_2^\omega$
- $\lim(L_1 \cdot L_2) \supseteq L_1 \cdot L_2^\omega$