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# 11. Exercises for the Course „Automata and Logic“

**Exercise 45:**

Let $\Sigma$ be an alphabet. Prove the following:

(a) If $L \subseteq \Sigma^+$ is regular, then there exists a finite automaton $\mathcal{A}$ with only **one** final state such that $L = L(\mathcal{A})$.

(b) If $L \subseteq \Sigma^*$ is regular, then there exists a finite automaton $\mathcal{A}$ with at most **two** final states such that $L = L(\mathcal{A})$.

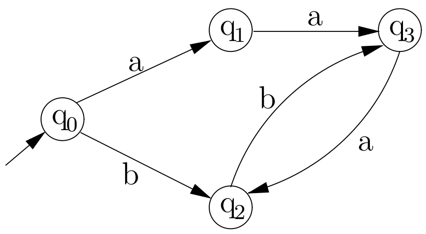
(c) There is **no** $k \in \mathbb{N}$ such that the following holds:

If $L \subseteq \Sigma^\omega$ is Büchi-recognizable, then there exists a Büchi-automaton $\mathcal{A}$ with at most $k$ final states such that $L = L_\omega(\mathcal{A})$.

**Hint**: For part (c), consider languages $a^\omega \cup b^\omega$, $a^\omega \cup b^\omega \cup c^\omega$, ...

**Exercise 46:**

Consider Büchi-automata based on the following transition system:




Check whether the recognized language is empty if the following states are selected as final states:

(a) $F = \{q_0, q_1\}$

(b) $F = \{q_2, q_3\}$

**Exercise 47:**

For a finite automaton $\mathcal{A}$, let $\mathcal{A}_{\text{det}}$ denote the minimal deterministic automaton such that $L(\mathcal{A}) = L(\mathcal{A}_{\text{det}})$.

Prove or refute the following:

(a) $\lim L(\mathcal{A}) = L_\omega(\mathcal{A}_{\text{det}})$

(b) $L_\omega(\mathcal{A}) \subseteq L_\omega(\mathcal{A}_{\text{det}})$

(c) $L_\omega(\mathcal{A}_{\text{det}}) \subseteq L_\omega(\mathcal{A})$

**Exercise 48:**

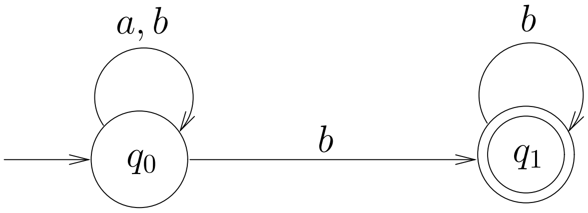
Let $(r_n)_{n\in\mathbf{N}}$ be a sequence of real numbers. Show that there exists an infinite sub-sequence of $(r_n)_{n\in\mathbf{N}}$ that is

- strictly increasing,
- strictly decreasing, or
- constant.

Hint: use Ramsey's theorem.

**Exercise 49:**

Let $\Sigma := \{a, b\}$ and $L \subseteq \Sigma^\omega$ be the $\omega$-language recognized by the following Büchi-automaton:




Use the method presented in the lecture to construct a Büchi-automaton that recognizes the language $\Sigma^\omega \setminus L$.