Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46,
01187 Dresden
Tel.: 0351/463–38237
Tel.: 0351/463–39171

13. Exercises for the Course „Automata and Logic“

Exercise 55:

A *Rabin-automaton* is a quintuple $\mathcal{A} := (Q, \Sigma, I, \Delta, \Omega)$, where Q, Σ, I, and Δ are defined as for non-deterministic Büchi-automata and $\Omega := \{(F_1, G_1), \ldots, (F_n, G_n)\}$ is a finite set of pairs (F_i, G_i) such that $F_i, G_i \subseteq Q$. For a word $\alpha \in \Sigma^\omega$, let $\mathbf{path}_\mathcal{A}(\alpha)$ denote the set of all paths in $\mathcal{A}$ labelled with α. For a path $p \in \mathbf{path}_\mathcal{A}(\alpha)$, let $\mathbf{inf}(p)$ denote the set of all states that are visited infinitely often. The ω-language $L_\omega(\mathcal{A})$ recognized by a Rabin-automaton is defined as

$$\{\alpha \in \Sigma^\omega \mid \exists i \in \{1, \ldots, n\}\ \exists p \in \mathbf{path}_\mathcal{A}(\alpha) : \mathbf{inf}(p) \cap F_i \neq \emptyset \text{ and } \mathbf{inf}(p) \cap G_i = \emptyset\}\,.$$

Show that every language recognized by a Rabin-automaton is also recognized by a Büchi-automaton by constructing, for a given Rabin-automaton $\mathcal{A}$, an S1S-formula $\varphi_\mathcal{A}$ defining the language $L_\omega(\mathcal{A})$.

Exercise 56:

Let $\Sigma = \Sigma_0 \cup \Sigma_1 \cup \Sigma_2$ be an alphabet with arity function, where $\Sigma_0 = \{x, y, z\}$, $\Sigma_1 = \{\neg\}$, and $\Sigma_2 = \{\wedge, \vee\}$. Define tree automata (either LR or RL) recognizing the tree-languages consisting of the following trees:

(a) trees containing the symbol $\vee$ exactly once;

(b) trees containg the symbol $\neg$ at least once on every path of the tree;

(c) trees describing *satisfiable* propositional formulas.

Exercise 57:

Let $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ be the non-deterministic LR-tree automaton defined as follows:

- $Q = \{0, 1\}$,
- $\Sigma = \{f, x, y\}$ with $\nu(f) = 2$ and $\nu(x) = \nu(y) = 0$,
- $I(x) = \{0, 1\}$ and $I(y) = \{0\}$,
- $\Delta_f(0, 0) = \{0\}$, $\Delta_f(0, 1) = \{1, 0\}$, $\Delta_f(1, 0) = \{1, 0\}$, and $\Delta_f(1, 1) = \{1\}$,
- $F = \{1\}$.

Do the following:

(a) Adapt the standard powerset construction from finite automata on words to LR-tree automata and use it to construct a deterministic LR-tree automaton $\mathcal{A}'$ such that $L(\mathcal{A}) = L(\mathcal{A}')$.

(b) Try to apply a similar construction to the RL-automaton $\mathcal{A}' = (Q', \Sigma, I', \Delta', F')$ (Example 6.10) with:

  - $Q' = \{q_0, q_1, q_x, q_y\}$,
  - $\Sigma = \{f, x, y\}$ with $\nu(f) = 2$ and $\nu(x) = \nu(y) = 0$,
  - $I' = \{q_0, q_1\}$,
  - $\Delta'_f(q_0) = \{(q_x, q_y)\}$, $\Delta'_f(q_1) = \{(q_y, q_x)\}$, $\Delta'_f(q_x) = \Delta'_f(q_y) = \emptyset$,
  - $F'(x) = \{q_x\}$, $F'(y) = \{q_y\}$.

  Explain why an adaptation of the powerset construction fails for RL-tree automata.

Exercise 58:

Let Σ be an alphabet with arity function, $x, y \in \Sigma_0$ und $U, V, W \subseteq \mathbf{T}_\Sigma$. Prove or refute:

(a) $U \cdot^x (V \cup W) = (U \cdot^x V) \cup (U \cdot^x W)$

(b) $(U \cdot^x V) \cdot^y W = U \cdot^x (V \cdot^y W)$

(c) $\left(U^{*,x}\right)^{*,y} = \left(U^{*,y}\right)^{*,x}$

Exercise 59:

Example 6.10 from the lecture shows that deterministic RL-tree automata recognize a smaller class of languages than non-deterministic ones. We call an RL-tree automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ *quasi-deterministic* if

- Δ is a deterministic transition assignment and
- $I \subseteq Q$ is a *set* of initial states.

Prove or refute:

(a) If $L \subseteq \mathbf{T}_\Sigma$ is a *finite* tree language, then there exists a quasi-deterministic tree automaton recognizing L.

(b) If $L \subseteq \mathbf{T}_\Sigma$ is a *recognizable* tree language, then there exists a quasi-deterministic tree automaton recognizing L.