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2. Exercises for the Course „Automata and Logic“

Exercise 6:

In the lecture, for a given deterministic finite automaton $\mathcal{A} = (Q, \Sigma, I, \delta, F)$ a relation $\sim_\mathcal{A} \subseteq Q \times Q$ is defined by setting

$$q \sim_\mathcal{A} q' \text{ iff } L_q(\mathcal{A}) = L_{q'}(\mathcal{A})$$

where $L_q(\mathcal{A})$ denotes the language recognized by the automaton $(Q, \Sigma, \{q\}, \delta, F)$.

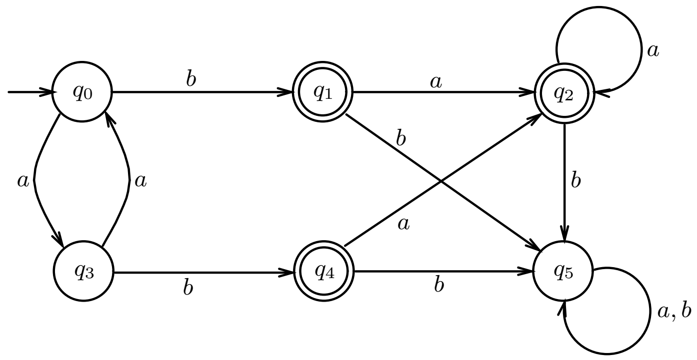
We inductively define a sequence of relations $\sim_0, \sim_1, \cdots$ as follows:

- $q \sim_0 q'$ iff $\{q, q'\} \subseteq F$ or $\{q, q'\} \cap F = \emptyset$;
- $q \sim_{i+1} q'$ iff $q \sim_i q'$ and $\delta(q, a) \sim_i \delta(q', a)$ for all $a \in \Sigma$.

Prove that, for every deterministic finite automaton $\mathcal{A}$, there exists an $n \in \mathbb{N}$ such that $\sim_n = \sim_\mathcal{A}$.

Exercise 7:

Let $\mathcal{A}$ be the finite automaton described by the following transition graph:



Construct the minimal automaton for $L(\mathcal{A})$.

Exercise 8:

The theorem of Nerode states that a language $L \subseteq \Sigma^*$ is regular if and only if the index of the Nerode right congruence $\sim_L$ (see Exercise 3) is finite, i.e. if there exist only finitely many equivalence classes of $\sim_L$. Use this theorem to show that the language

$$\{a^n b^n \mid n \geq 0\}$$

is *not* regular.

Exercise 9:

Let $M := \{1, m\}$ with $1 \neq m$. Determine all functions $\circ$ such that $(M, \circ, 1)$ is a monoid.

Exercise 10:

Let $(M, \circ, 1)$ be a monoid. A relation $R \subseteq M \times M$ is called a *congruence relation* if it is an equivalence relation and, for all $x, y, x', y' \in M$, xRy and $x'Ry'$ implies $(x \circ x')\ R\ (y \circ y')$.

Consider the monoid $(\mathbb{Z}, +, 0)$ and the following relations on $\mathbb{Z}$, where we write $3|z$ to express that z is divided by 3 without remainder:

(a) $z_1 R_1 z_2$ iff $3|(z_1 - z_2)$;

(b) $z_1 R_2 z_2$ iff either $3|z_1$ and $3|z_2$ or $3\nmid z_1$ and $3\nmid z_2$.

For $z \in \mathbb{Z}$, we use $[z]_i$ to denote the equivalence class of z w.r.t. the relation R_i. For $i \in \{1, 2\}$, define the monoid $(M_i, \circ_i, 1_i)$ by setting

- $M_i := \{[z]_i \mid z \in \mathbb{Z}\}$;
- $[z]_i \circ_i [z']_i := [z + z']_i$;
- $1_i := 1$.

Prove the following:

(a) R_1 and R_2 are equivalence relations;

(b) R_1 is a congruence relation, but R_2 is not;

(c) $(M_1, \circ_1, 1_1)$ is well-defined, but $(M_2, \circ_2, 1_2)$ is not.