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4. Exercises for the Course „Automata and Logic“

Exercise 16:

Let Σ be an alphabet, $L \subseteq \Sigma^*$ a regular language, M_L the syntactic monoid of L, and Q_L the set of states of a minimal automaton accepting L.

(a) Show that $|M_L| \geq |Q_L|$.

(b) Let $V := \{(M, \circ, 1) \mid (M, \circ, 1) \text{ is a monoid with } |M| = 2\}$ be the class of all monoids with two elements. For $\Sigma := \{a\}$, determine the class $L(V)_\Sigma$ of regular languages such that $M_L \in V$.

(c) Determine the alphabets Σ for which the following equivalence holds:

$$\text{for all regular languages } L \subseteq \Sigma^*, \text{ we have } |M_L| = 2 \text{ iff } |Q_L| = 2$$

Exercise 17:

Show that the class of all finite groups is an M-variety by proving that this class is closed under building submonoids, binary direct products, and homomorphic images.

Exercise 18:

Let V be an M-variety. Show that $L(V)_\Sigma$ is closed under union without using Theorem 1.22 from the lecture.

Exercise 19:

Let Σ be an alphabet. Prove or refute the following claims:

(a) Every regular language $L \subseteq \Sigma^*$ is accepted by its syntactic monoid.

(b) If $L \subseteq \Sigma^*$ is accepted by a finite group, then the syntactic monoid of L is a finite group.

(c) For every regular language $L \subseteq \Sigma^*$, the syntactic monoid M_L is the smallest monoid accepting L, i.e. for all monoids M accepting L we have $|M_L| \leq |M|$.

(d) Let $\overleftarrow{w}$ denote the mirror image of the word w, i.e. for $w = a_1 \dots a_n$ we have $\overleftarrow{w} := a_n \dots a_1$. Moreover, for a language $L \subseteq \Sigma^*$, we define $\overleftarrow{L} := \{\overleftarrow{w} \mid w \in L\}$. Claim: if the minimal automaton for L has n states, then the minimal automaton for $\overleftarrow{L}$ has n states.