Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nthnitzer Str. 46,
01187 Dresden
Tel.: 0351/463–38237
Tel.: 0351/463–39171

5. Exercises for the Course „Automata and Logic“

Exercise 20:

Let $(M, \circ, 1)$ be a monoid with two right absorbing elements, i.e. we have $M := \{1, a, b\}$ with operation $\circ$ defined as

$\circ$	1	a	b
1	1	a	b
a	a	a	b
b	b	a	b

Prove that there exists a regular language L whose syntactic monoid is isomorphic to $(M, \circ, 1)$.

Exercise 21:

Let V be the M-variety of all commutative finite groups. Show that, for the language

$$L = \{a^n \mid n \text{ is even}\},$$

it holds that $L \in L(V)_{\{a\}}$, but $L \notin L(V)_{\{a,b\}}$.

Exercise 22:

Determine a language $L \subseteq \{a, b\}^*$ such that the syntactic semi-group S_L and the syntactic monoid M_L are isomorphic.

Exercise 23:

For each of the following words over the alphabet $\{0, 1\}^k$, give a corresponding interpretation over the predicate symbols $P_1, \ldots, P_k$ as discussed in the lecture:

for $k = 2$: $(1, 1), (1, 1), (1, 1), (0, 0)$
for $k = 3$: $(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)$
for $k = 3$: $(1, 1, 0), (1, 0, 1), (0, 1, 0), (1, 1, 1)$

Describe all interpretations that correspond to words of the language $L(((0, 1) \cdot (1, 0))^+) \subseteq (\{0, 1\}^2)^+$.

Exercise 24:

Let $\Sigma := \{a, b\}$. For each of the following regular expressions r_i, give a first-order formula φ_i such that $L(r_i) = L(\varphi_i)$:

(a) $r_1 = \Sigma^*$;

(b) $r_2 = \epsilon$;

(c) $r_3 = (abb^*)^*$;

(d) $r_4 = a^*b^* + b^*a^*$;

(e) $r_5 = (aaa \cdot \Sigma^*) + b^*$.

For each of the following formulas φ_i, give a regular expression r_i such that $L(\varphi_i) = L(r_i)$:

(a) $\varphi_1 = \exists x, y, z \,.\, x < y \wedge y < z;$

(b) $$\begin{aligned}\varphi_2 = {} & Q_b(\min) \\ & \wedge \forall x \,.\, \big(Q_b(x) \to (\exists y \,.\, \mathrm{succ}(x,y) \wedge Q_a(y))\big) \\ & \wedge \forall x \,.\, \big(Q_a(x) \to (\forall y \,.\, \mathrm{succ}(x,y) \to Q_b(y))\big);\end{aligned}$$

(c) $\varphi_3 = \forall x \,.\, \big(Q_a(x) \to \exists y \,.\, (x < y \wedge Q_b(y))\big);$

(d) $$\begin{aligned}\varphi_4 = \forall x \,.\, \big(& \max(x) \\ & \vee \exists y \,.\, \mathrm{succ}(x,y) \wedge \max(y) \\ & \vee Q_b(x) \to \forall y . y < x \to Q_b(y)\big).\end{aligned}$$

(e) $$\begin{aligned}\varphi_5 = {} & \forall x \,.\, \big(Q_a \to \exists y \,.\, \mathrm{succ}(x,y) \wedge Q_b(y))\big) \\ & \wedge \forall x \,.\, \big(Q_b \to \exists y \,.\, \mathrm{succ}(x,y) \wedge Q_a(y))\big)\end{aligned}$$