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8. Exercises for the Course „Automata and Logic“

Exercise 35:

Let $\Sigma = \{a\}$. As in the lecture, let $L_{k,n}$ denote the set of all first-order formulas (over the non-logical symbols $=$, $<$, and Q_a) containing k free variables and having quantifier depth at most n. For the following combinations of k and n, determine a *finite* set $\Gamma_{k,n}$ such that, for each formula $\varphi \in L_{k,n}$, there exists a $\psi \in \Gamma_{k,n}$ with $\varphi \equiv \psi$.

(a) $k = 1, n = 0$;

(b) $k = 2, n = 0$;

(c) $k = 0, n = 1$;

(d) $k = 1, n = 1$;

Exercise 36:

For the combinations of k and n listed in the previous exercise, determine the equivalence classes of the relations $\equiv_{k,n}$. To do this, use the sets $\Gamma_{k,n}$ determined in the previous exercise.

Exercise 37:

Using the results from the previous exercise, give the formulas ϕ_W for each equivalence class W of $\equiv_{2,0}$. Then determine a finite disjunction of formulas ϕ_W for $\equiv_{k,n}$-classes W which is equivalent to the following formulas:

(a) true

(b) $\neg(x < y) \vee x = y$

(c) false