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11. Exercises for the lecture **Term Rewriting Systems**

Exercise 44:

For each of the following pairs (s, t) of terms, check whether $s \trianglerighteq_{emb} t$.
(a) $f(x) \trianglerighteq_{emb} a$? (b) $f(b) \trianglerighteq_{emb} a$? (c) $g(g(x, y), g(a, f(z))) \trianglerighteq_{emb} g(y, g(a, z))$?

Exercise 45:

Prove the following claims:

(a) The reduction relation $\xrightarrow{*}_{emb}$ given by the TRS

$$R_{emb} := \{f(x_1, \ldots, x_n) \longrightarrow x_i \mid n \geq 1,\ f \in \Sigma^{(n)} \text{ und } 1 \leq i \leq n\}$$

and the homeomorphic embedding $\trianglerighteq_{emb}$ are identical, i.e., $s \xrightarrow{*}_{emb} t$ iff $s \trianglerighteq_{emb} t$.

(b) $\trianglerighteq_{emb}$ is a partial order.

(c) $\trianglerighteq_{emb}$ is well-founded (prove this last claim without using Kruskall's Theorem).

Exercise 46:

In the proof of Theorem 5.32, we have used that $\trianglerighteq_{emb}$ is a well-partial-order. Explain why $\trianglerighteq_{emb}$ being a well-founded partial order would not have been sufficient.

Exercise 47:

Prove that the TRS $R := \{f(f(x)) \longrightarrow f(g(f(x)))\}$ is terminating.