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15. Exercises for the lecture **Term Rewriting Systems**

Exercise 61:

The semi-decision procedure described in the proof of Theorem 7.7 of the lecture is rather inefficient: for the input $s \approx_E t$, all R_i-normal forms of s and t are computerd in the i'th iteration of the "Repeat"-loop. Show that the following modification of the procedure still yields a partial decision procedure for the word problem:

- Begin with $s_0 := s$ and $t_0 := t$.
- After the ith "Repeat"-loop, compute *one arbitrary* R_i-normal form s_i of s_{i_1} and *one arbitrary* R_i-normal form t_i of t_{i_1}.
- Output "yes" ($s \approx_E t$) iff there exists an n such that $s_n \approx t_n$.

Hint: show: (1) for all $n \geq 0$ it holds that $s \approx_E s_n$ and $t \approx_E t_n$; (2) since R_∞ is terminating, there exists an n such that $s_n = s_m$ and $t_n = t_m$ for all $m \geq n$.

Exercise 62:

(a) Consider the following set of identities:

$$E := \{f(f(x, y), z) \approx f(x, f(y, z)),\ f(x, x) \approx x,\ f(f(x, y), x) \approx x\}$$

Apply the rules of the improved completion procedure to E. Use a strategy that resembles the basic completion procedure, but simplifies rules as follows: upon adding new rules, simplify the old ones by means of L-Simplify-Rule and R-Simplify-Rule.

(b) Consider the following set of identities:

$$E := \{x + (y + z) \approx (x + y) + z,\ f(x) + f(y) \approx f(x + y)\}$$

Apply the completion procedure described above to the input E and the polynomial order induced by

$$P_f(X) = X + 1,\ P_+(X, Y) = XY^2$$

Exercise 63:

Consider the following completion procedure for ground term rewriting systems:

Input	G_0, a finite set of ground identities over Σ, $>$, a reduction order total on the set of ground terms over Σ.
Procedure	Apply the rules L-Simplify-Rule, Delete and Orient, until no rule is applicable any more.
Output	a ground term rewriting system.

Show that this procedure: (a) always terminates,
(b) is fair,
(c) is correct and
(d) never ends in failure.