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6. Exercises for the lecture **Term Rewriting Systems**

Exercise 22:

First, describe in your own words what $s \underset{E}{\longrightarrow} t$ means.
Next, consider the following set E of identities

$$E = \{ \begin{array}{rcl@{\qquad}rcl} u(x,y) & \approx & u(y,x), & i(x,y) & \approx & i(y,x), \\ u(x,i(y,x)) & \approx & x, & i(x,u(y,x)) & \approx & x\} \end{array}$$

and the definition of the relation $\underset{E}{\longrightarrow}$, and present a reduction $u(i(y,i(u(x,y),x)),y) \underset{E}{\overset{*}{\longrightarrow}} y$.

Exercise 23:

Prove Lemma 3.10 from the lecture:
Lemma 3.10 Let E be a set of identities and $\underset{E}{\longrightarrow}$ the induced reduction relation. Then $\underset{E}{\longrightarrow}$ is closed under substitutions and compatible with Σ-contexts.

Exercise 24:

Prove Proposition 3.16 from the lecture:
Proposition 3.16 Let $\mathcal{A}$ be a Σ-algebra and $\varphi : X \to A$ a mapping (recall that A is the carrier set of $\mathcal{A}$). Then there exists a *unique homomorphism* $\Phi : \mathcal{T}(\Sigma, X) \to \mathcal{A}$ that extends φ (i.e., with $\varphi(x) = \Phi(x)$ for all $x \in X$).

Exercise 25:

Let ρ and τ be substitutions.

(a) Describe the variable range $\mathcal{V}\mathsf{Ran}(\rho\tau)$ (see Definition 3.6)

(b) Under what conditions do we have $\rho\tau = \tau$?

(c) Under what conditions do we have $\tau\tau = \tau$?

Exercise 26:

Explain, in your own words, Birkhoff's Theorem and why it is of importance.