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7. Exercises for the lecture **Term Rewriting Systems**

Exercise 27:

Let E be a set of identities with some $\ell \approx r \in E$ such that

- ℓ is a variable or
- $\mathcal{V}ar(r) \not\subseteq \mathcal{V}ar(\ell)$.

Prove that $\xrightarrow[E]{}$ is not terminating.

Exercise 28:

Let $G_1 := \{f^3(a) \approx a,\ f^5(a) \approx a\}$ and $G_2 := \{f^4(a) \approx a,\ f^6(a) \approx a\}$. Consider the congruence closures of G_1 and G_2 and verify whether $f^2(a) \approx a$ holds in G_1 or G_2.

Exercise 29:

Let G be a set of ground identities and $CC(G)$ the congruence closure of G. Prove that

$$\xrightarrow[G]{} \subseteq CC(G).$$

(This implies that $\xleftrightarrow[G]{*} \subseteq CC(G)$ and completes the proof of Lemma 4.8 of the lecture.)

Exercise 30:

Let u, x, y, z be variables. Use the unification algorithm recalled in the lecture to solve the following two unification problems:

(a) $S_1 := \{f(h(x), g(x,u)) =^? f(z, g(f(y,y), z))\}$,

(b) $S_2 := \{h(x, g(x,y), y) =^? h(x, g(a,y), y), \quad z =^? h(x, g(x,b), b)\}$.

Exercise 31:

From the unification algorithm recalled in the lecture, design a direct decision procedure for the matching problem. **Hint:** "direct" means that no constants are introduced in the right term. Instead, the rules are to be modified such that the new algorithm returns "the input terms do not match" or a matcher for the input terms as soon as possible.