Lemma 1.4 Let $\phi$ be a pinpointing formula for $T$ w.r.t. $A \sqsubseteq_T B$. If valuations are ordered by set inclusion, then

$$M := \{ T_{\mathcal{V}} \mid \mathcal{V} \text{ is a minimal valuation satisfying } \phi \}$$

is the set of all MinAs for $T$ w.r.t. $A \sqsubseteq_T B$.

Proof. The mapping $\text{lab} : T \rightarrow \text{lab}(T)$ is bijective. Thus, its extension to the powersets

$$\text{lab} : \mathcal{P}(T) \rightarrow \mathcal{P}(\text{lab}(T)) : S \mapsto \text{lab}(S)$$

is also bijective (and has as its inverse the mapping $T_{\bullet} : \mathcal{V} \mapsto T_{\mathcal{V}}$). lab is even an isomorphism between the posets $(\mathcal{P}(\text{lab}(T)), \subseteq)$ and $(\mathcal{P}(T), \subseteq)$, because of the monotonicity of the image of any mapping.

If we set $C := \{ S \subseteq T \mid A \sqsubseteq_S B \}$ and $D := \{ \mathcal{V} \subseteq \text{lab}(T) \mid \mathcal{V} \text{ satisfies } \phi \}$, the restriction of lab to $C$ is still an isomorphism from $C$ to $D$. This follows from the assumption that $\phi$ is a pinpointing formula for $T$ w.r.t. $A \sqsubseteq_T B$, i.e. for every $S \subseteq T$ we have that $A \sqsubseteq_S B$ iff $\text{lab}(S)$ satisfies $\phi$.

Thus, the sets of minimal elements of $C$ and $D$ are mapped into each other by lab and $T_{\bullet}$:

$$M = \{ T_{\mathcal{V}} \mid \mathcal{V} \subseteq \text{lab}(T) \text{ is a minimal valuation satisfying } \phi \}$$

$$= \{ T_{\text{lab}(S)} \mid S \subseteq T \text{ is a minimal axiom set with } A \sqsubseteq_S B \}$$

$$= \{ S \mid S \subseteq T \text{ is a minimal axiom set with } A \sqsubseteq_S B \}$$

$\square$