Lemma 1.4 *Let ϕ be a pinpointing formula for $\mathcal{T}$ w.r.t. $A \sqsubseteq_{\mathcal{T}} B$. If valuations are ordered by set inclusion, then*

$$M := \{\mathcal{T}_{\mathcal{V}} \mid \mathcal{V} \textit{ is a minimal valuation satisfying } \phi\}$$

is the set of all MinAs for $\mathcal{T}$ w.r.t. $A \sqsubseteq_{\mathcal{T}} B$.

Proof. The mapping $\mathrm{lab} : \mathcal{T} \to \mathrm{lab}(\mathcal{T})$ is bijective. Thus, its extension to the powersets

$$\mathrm{lab} : \mathcal{P}(\mathcal{T}) \to \mathcal{P}(\mathrm{lab}(\mathcal{T})) : \mathcal{S} \mapsto \mathrm{lab}(\mathcal{S})$$

is also bijective (and has as its inverse the mapping $\mathcal{T}_{\bullet} : \mathcal{V} \mapsto \mathcal{T}_{\mathcal{V}}$). lab is even an isomorphism between the posets $(\mathcal{P}(\mathrm{lab}(\mathcal{T})), \subseteq)$ and $(\mathcal{P}(\mathcal{T}), \subseteq)$, because of the monotonicity of the image of any mapping.

If we set $\mathcal{C} := \{\mathcal{S} \subseteq \mathcal{T} \mid A \sqsubseteq_{\mathcal{S}} B\}$ and $\mathcal{D} := \{\mathcal{V} \subseteq \mathrm{lab}(\mathcal{T}) \mid \mathcal{V} \text{ satisfies } \phi\}$, the restriction of lab to $\mathcal{C}$ is still an isomorphism from $\mathcal{C}$ to $\mathcal{D}$. This follows from the assumption that ϕ is a pinpointing formula for $\mathcal{T}$ w.r.t. $A \sqsubseteq_{\mathcal{T}} B$, i.e. for every $\mathcal{S} \subseteq \mathcal{T}$ we have that $A \sqsubseteq_{\mathcal{S}} B$ iff $\mathrm{lab}(\mathcal{S})$ satisfies ϕ.

Thus, the sets of minimal elements of $\mathcal{C}$ and $\mathcal{D}$ are mapped into each other by lab and $\mathcal{T}_{\bullet}$:

$$\begin{aligned} M &= \{\mathcal{T}_{\mathcal{V}} \mid \mathcal{V} \subseteq \mathrm{lab}(\mathcal{T}) \text{ is a minimal valuation satisfying } \phi\} \\ &= \{\mathcal{T}_{\mathrm{lab}(\mathcal{S})} \mid \mathcal{S} \subseteq \mathcal{T} \text{ is a minimal axiom set with } A \sqsubseteq_{\mathcal{S}} B\} \\ &= \{\mathcal{S} \mid \mathcal{S} \subseteq \mathcal{T} \text{ is a minimal axiom set with } A \sqsubseteq_{\mathcal{S}} B\} \end{aligned}$$

$\square$