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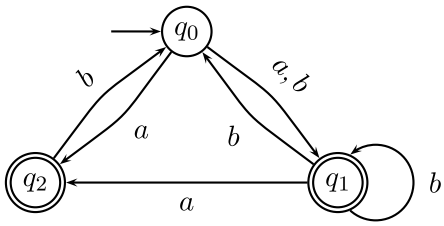
1. Exercises for the Course 'Automata and Logic'

Exercise 1:

Let $\Sigma = \{a, b\}$ be an alphabet and $\alpha := a^+b^* + b^+a^*$ a regular expression over Σ. Give a regular expression β for the complement language of α (that is, β should describe the set of words over Σ that are not expressed by α).

Exercise 2:

Let the non-deterministic automaton $\mathcal{A} := (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_1, q_2\})$ be given by the following transition system:



Apply the power set construction to $\mathcal{A}$ in order to obtain a *deterministic* automaton that accepts the same language as $\mathcal{A}$.

Exercise 3:

For a language $L \subseteq \Sigma^*$, the *Nerode right congruence* ρ_L is defined as follows: for $u, v \in \Sigma^*$, $u\rho_L v$ iff for all $w \in \Sigma^*$ it holds that

$$uw \in L \Leftrightarrow vw \in L.$$

Let $\mathcal{A}_L := (Q_L, \Sigma, q_L, \delta_L, F_L)$ be a deterministic automaton where:

- $Q_L := \{[u] \mid u \in \Sigma^*\}$ where $[u] := \{v \in \Sigma^* \mid u\rho_L v\}$,
- $q_L := [\varepsilon]$ for ε the empty word,
- $\delta_L([u], a) := [ua]_{\rho_L}$ for $u \in \Sigma^*, a \in \Sigma$,
- $F_L := \{[u] \mid u \in L\}$.

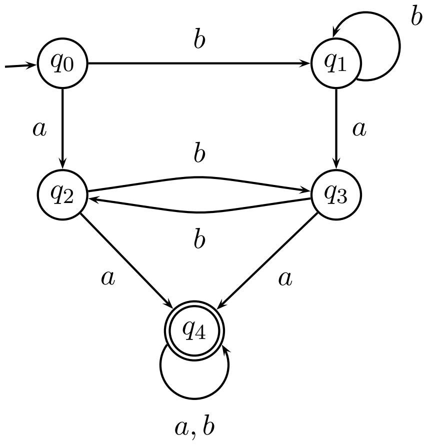
Show the following:

(a) $\mathcal{A}_L$ is well-defined

(b) $\mathcal{A}_L$ is minimal; i.e. for every deterministic automaton $\mathcal{A} := (Q, \Sigma, q_0, \delta, F)$ with $L(\mathcal{A}) = L$, it holds that $|Q_L| \leq |Q|$.

Exercise 4:

Let $\mathcal{A}$ be the automaton accepting words over the alphabet $\Sigma := \{a, b\}$, described by the following transition system:



Construct an automaton $\mathcal{A}'$ such that $L(\mathcal{A}') = L(\mathcal{A})$ and $\mathcal{A}'$ is minimal.

Exercise 5:

Prove the following by giving a decision procedure:

(a) The *emptiness problem* for regular languages is decidable.

(b) The *inclusion problem* for regular languages is decidable.