1. Exercises for the Course
‘Automata and Logic’

Exercise 1:
Let $\Sigma = \{a, b\}$ be an alphabet and $\alpha := a^*b + b^*a^*$ a regular expression over $\Sigma$. Give a regular expression $\beta$ for the complement language of $\alpha$ (that is, $\beta$ should describe the set of words over $\Sigma$ that are not expressed by $\alpha$).

Exercise 2:
Let the non-deterministic automaton $A := (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_1, q_2\})$ be given by the following transition system:

Apply the power set construction to $A$ in order to obtain a deterministic automaton that accepts the same language as $A$.

Exercise 3:
For a language $L \subseteq \Sigma^*$, the Nerode right congruence $\rho_L$ is defined as follows: for $u, v \in \Sigma^*$, $u \rho_L v$ iff for all $w \in \Sigma^*$ it holds that

$$uw \in L \Leftrightarrow vw \in L.$$ 

Let $A_L := (Q_L, \Sigma, q_L, \delta_L, F_L)$ be a deterministic automaton where:

- $Q_L := \{[u] \mid u \in \Sigma^*\}$ where $[u] := \{v \in \Sigma^* \mid u \rho_L v\}$,
- $q_L := [\varepsilon]$ for $\varepsilon$ the empty word,
- $\delta_L([u], a) := [ua]_{\rho_L}$ for $u \in \Sigma^*, a \in \Sigma$,
- $F_L := \{[u] \mid u \in L\}$.

Show the following:

(a) $A_L$ is well-defined

(b) $A_L$ is minimal; i.e. for every deterministic automaton $A := (Q, \Sigma, q_0, \delta, F)$ with $L(A) = L$, it holds that $|Q_L| \leq |Q|$.
Exercise 4:
Let $A$ be the automaton accepting words over the alphabet $\Sigma := \{a, b\}$, described by the following transition system:

Construct an automaton $A'$ such that $L(A') = L(A)$ and $A'$ is minimal.

Exercise 5:
Prove the following by giving a decision procedure:

(a) The emptiness problem for regular languages is decidable.

(b) The inclusion problem for regular languages is decidable.