10. Exercises for the Course
‘Automata and Logic’

Exercise 47:
Let $\Sigma := \{a, b\}$ and $L \subseteq \Sigma^\omega$ be the $\omega$-language recognized by the following Büchi automaton:

![Büchi automaton diagram]

Use the method presented in the lecture to construct a Büchi-automaton that recognizes the language $\Sigma^\omega \setminus L$.

Exercise 48:
Prove that, for every $\omega$-regular language $L$ there is a Büchi automaton $A$ with $L_\omega(A) = L$, such that from every state $q$ of $A$ there are at most two transitions using the same alphabet symbol.

Exercise 49:
Let $\Sigma := \{a, b, c\}$. From the transition system

![Transition system diagram]

we derive four Muller automata $A_1, A_2, A_3$ and $A_4$ by selecting the sets of final states $F_1, F_2, F_3, F_4$ as follows:

(a) $F_1 = \{\{q_0, q_3\}, \{q_3\}\}$,
(b) $F_2 = \{\{q_0, q_1\}, \{q_2\}\}$,
(c) $F_3 = \{\{q_0, q_1, q_2\}\}$, and
(d) $F_4 = \{\{q_0\}, \{q_0, q_1\}, \{q_2\}, \{q_0, q_1, q_2\}\}$.

Determine the $\omega$-languages $L_\omega(A_1), L_\omega(A_2), L_\omega(A_3)$, and $L_\omega(A_4)$. 
Exercise 50:
For each of the following languages $L_i$, give an S1S-formula $\varphi_i$ such that $L_\omega(\varphi_i) = L_i$:

(a) $L_1 = (abb^*)^\omega$;

(b) $L_2 = ((aa)^+ (bb)^+)^\omega$;

(c) $L_3 = (aaa)^+ b(a + b)^\omega$.

Exercise 51:
Transform the S1S-formula $P(0)$ into an equivalent S1S$_0$-formula.