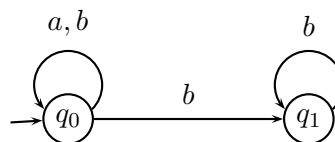


10. Exercises for the Course ‘Automata and Logic’

Exercise 47:

Let $\Sigma := \{a, b\}$ and $L \subseteq \Sigma^\omega$ be the ω -language recognized by the following Büchi automaton:



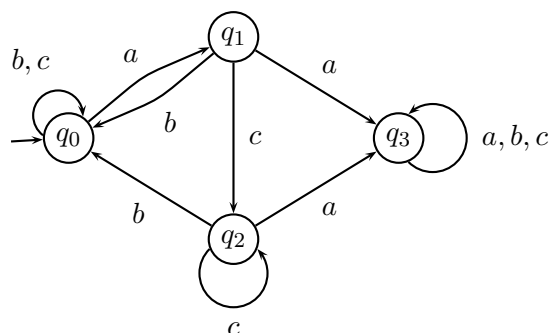
Use the method presented in the lecture to construct a Büchi-automaton that recognizes the language $\Sigma^\omega \setminus L$.

Exercise 48:

Prove that, for every ω -regular language L there is a Büchi automaton \mathcal{A} with $L_\omega(\mathcal{A}) = L$, such that from every state q of \mathcal{A} there are **at most two** transitions using the same alphabet symbol.

Exercise 49:

Let $\Sigma := \{a, b, c\}$. From the transition system



we derive four Muller automata $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ and \mathcal{A}_4 by selecting the sets of final states $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4$ as follows:

- (a) $\mathcal{F}_1 = \{\{q_0, q_3\}, \{q_3\}\}$,
- (b) $\mathcal{F}_2 = \{\{q_0, q_1\}, \{q_2\}\}$,
- (c) $\mathcal{F}_3 = \{\{q_0, q_1, q_2\}\}$, and
- (d) $\mathcal{F}_4 = \{\{q_0\}, \{q_0, q_1\}, \{q_2\}, \{q_0, q_1, q_2\}\}$.

Determine the ω -languages $L_\omega(\mathcal{A}_1), L_\omega(\mathcal{A}_2), L_\omega(\mathcal{A}_3)$, and $L_\omega(\mathcal{A}_4)$.

Exercise 50:

For each of the following languages L_i , give an S1S-formula φ_i such that $L_\omega(\varphi_i) = L_i$:

(a) $L_1 = (abb^*)^\omega$,

(b) $L_2 = ((aa)^+(bb)^+)^\omega$,

(c) $L_3 = (aaa)^+b(a+b)^\omega$.

Exercise 51:

Transform the S1S-formula $P(0)$ into an equivalent S1S₀-formula.