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11. Exercises for the Course 'Automata and Logic'

Exercise 52:

Let $L := (a^+b)^\omega \cup (b^+a)^\omega$. **Use the proof of Theorem 5.4** to construct a closed S1S-formula φ with $L_\omega(\varphi) = L$.

Exercise 53:

A *Rabin-automaton* is a tuple $\mathcal{A} := (Q, \Sigma, I, \Delta, \Omega)$, where Q, Σ, I, and Δ are defined as for non-deterministic Büchi automata and $\Omega := \{(F_1, G_1), \ldots, (F_n, G_n)\}$ is a finite set of pairs (F_i, G_i) such that $F_i, G_i \subseteq Q$. For a word α, $\mathbf{path}_\mathcal{A}(\alpha)$ denotes the set of all paths in $\mathcal{A}$ labelled with α. For a path $p \in \mathbf{path}_\mathcal{A}(\alpha)$, let $\mathbf{inf}(p)$ denote the set of all states that are visited infinitely often. The ω-language $L_\omega(\mathcal{A})$ recognized by a Rabin automaton is defined as

$$\{\alpha \in \Sigma^\omega \mid \exists i \in \{1, \ldots, n\} \exists p \in \mathbf{path}_\mathcal{A}(\alpha) : \mathbf{inf}(p) \cap F_i \neq \emptyset \text{ and } \mathbf{inf}(p) \cap G_i = \emptyset\}.$$

Show thatevery language recognized by a Rabin-automaton is also recognized by a Büchi-automaton by constructing, for a given Rabin-automaton $\mathcal{A}$, an S1S-formula $\varphi_\mathcal{A}$ defining the language $L_\omega(\mathcal{A})$.

Exercise 54:

Let $\Sigma = \Sigma_0 \cup \Sigma_1 \cup \Sigma_2$ be an alphabet with arity function, where $\Sigma_0 = \{x, y, z\}, \Sigma_1 = \{\neg\}$, and $\Sigma_2 = \{\wedge, \vee\}$. Define tree automata (either LR or RL) recognizing the tree-languages consisting of the following trees:

(a) trees containing the symbol $\vee$ exactly once;

(b) trees containing the symbol $\neg$ at least once on every path of the tree;

(c) trees describing *satisfiable* propositional formulas.

Exercise 55:

Let $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ be the non-deterministic LR-tree automaton given by:

- $Q = \{0, 1\}$,
- $\Sigma = \{f, x, y\}$ with $\nu(f) = 2$ and $\nu(x) = \nu(y) = 0$,
- $I(x) = \{0, 1\}$ and $I(y) = \{0\}$,
- $\Delta_f(0, 0) = \{0\}, \Delta_f(0, 1) = \{1, 0\}, \Delta_f(1, 0) = \{1, 0\}$, and $\Delta_f(1, 1) = \{1\}$,
- $F = \{1\}$.

Do the following:

(a) Adapt the standard powerset construction from finite automata on words to LR-tree automata and use it to construct a deterministic LR-tree automaton $\mathcal{A}'$ such that $L(\mathcal{A}) = L(\mathcal{A}')$.

(b) Try to apply a similar construction to the RL-automaton from Example 6.10. Explain why this method fails for RL-automata.

Exercise 56:

Let Σ be an alphabet with arity function, $x, y \in \Sigma_0$ and $U, V, W \subseteq \mathbf{T}_\Sigma$. Prove or refute:

- $U \cdot^x (V \cup W) = (U \cdot^x V) \cup (U \cdot^x W)$,
- $(U \cdot^x V) \cdot^y W = U \cdot^x (V \cdot^y W)$,
- $(U^{*,x})^{*,y} = (U^{*,y})^{*,x}$.