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12. Exercises for the Course 'Automata and Logic'

Exercise 57:

Example 6.10 from the lecture shows that deterministic RL-tree automata recognize a smaller class of languages than non-deterministic ones. We call an RL-tree automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ *quasi-deterministic* if

- Δ is a deterministic transition assignment, and
- $I \subseteq Q$ is a *set* of initial states.

Prove or refute:

(a) If $L \subseteq \mathbf{T}_\Sigma$ is a *finite* tree language, then there exists a quasi-deterministic tree automaton recognizing L,

(b) If $L \subseteq \mathbf{T}_\Sigma$ is a *recognizable* tree language, then there exists a quasi-deterministic tree automaton recognizing L.

Exercise 58:

Construct regular expressions for the following tree languages:

(a) The language of all trees that represent arithmetic expressions over the binary symbols $+$ and $\cdot$, the unary symbol $-$, and the variables (i. e. nullary symbols) x, y, and z.

(b) The language of all trees that represent regular expressions over the alphabel $\Sigma_L := \{a, b\}$.

You may add auxiliary symbols to the alphabets if necessary.

Exercise 59:

Let Σ be an alphabet with arity function. Show the following **without using Proposition 6.20**:

(a) For every $t \in \mathbf{T}_\Sigma$, the language $\{t\}$ is recognizable.

(b) For every $t \in \mathbf{T}_\Sigma$, the language $\{t\}$ is regular.

Exercise 60:

Let $\Sigma = \{\vee, \wedge, \neg, \top, \bot\}$ be an alphabet with the obvious arity function. The following RL-tree automaton $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ accepts precisely those trees that represent Boolean expressions evaluating to "false":

- $Q = \{0, 1\}$,
- $I = \{0\}$,
- $\Delta_{\neg}(0) = \{1\}, \Delta_{\neg}(1) = \{0\}$
- $\Delta_{\wedge}(0) = \{(0,0), (0,1), (1,0)\}, \Delta_{\wedge}(1) = \{(1,1)\}$
- $\Delta_{\vee}(0) = \{(0,0)\}, \Delta_{\vee}(1) = \{(0,1), (1,0), (1,1)\}$
- $F(\top) = 1, F(\bot) = 0.$

Use the method from the proof of Theorem 6.20 to construct a regular expression for the tree language recognized by $\mathcal{A}$.

Exercise 61:

Devise a quadratic time algorithm that decides the emptiness problem for LR-tree automata.