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13. Exercises for the Course 'Automata and Logic'

Exercise 62:

Let $\mathcal{A} = (Q, \Sigma, I, \Delta, F)$ be an RL-tree automaton given by:

- $Q = \{1, \ldots, 4\}$,
- $\Sigma = \{g, n, a, b\}$ with $\nu(g) = 2, \nu(n) = 1$, and $\nu(a) = \nu(b) = 0$,
- $I = \{1\}$,
- $\Delta_n(1) = \{1\}, \Delta_n(2) = \{3\}, \Delta_n(3) = \{1, 2\}, \Delta_n(4) = \{1, 3\}$,
- $\Delta_g(1) = \{(1, 1), (1, 2), (3, 4), (4, 1)\}$,
- $\Delta_g(2) = \emptyset$,
- $\Delta_g(3) = Q \times Q$,
- $\Delta_g(4) = \{(1, 2), (1, 4), (2, 4), (2, 2)\}$,
- $F(a) = \{2\}, F(b) = \{2, 3\}$.

Decide whether $L(\mathcal{A}) = \emptyset$ or not.

Exercise 63:

Let $\Sigma = \{a, b\}$ be an alphabet with two binary symbols and

$$L = \{t \in \mathbf{T}_\Sigma^\omega \mid \text{ there is a path in } t \text{ containing only finitely many } a\text{'s}\}.$$

(a) Is L Rabin-recognizable?

(b) Is L Büchi-recognizable?

Exercise 64:

Let Σ be an alphabet with arity function containing at least two binary symbols f and g. Prove or refute that the ω-tree language $L = \{f(t, t) \mid t \in \mathbf{T}_\Sigma^\omega\}$ is Büchi-recognizable.

Exercise 65:

Complete the proof of Theorem 7.11 by showing the following:
Let Σ be an alphabet of binary symbols and $L \subseteq \mathbf{T}^{\omega}_{\Sigma}$ be such that there exists a set $F = \{f_1, \ldots, f_m\}$ of nullary symbols and recognizable tree-languages $L_0, \ldots, L_m \subseteq \mathbf{T}_{\Sigma \cup F}$ such that

$$L = L_0 \cdot^{(f_1,\ldots,f_m)} (L_1, \ldots, L_m)^{\omega,(f_1,\ldots,f_m)}.$$

Then L is Büchi-recognizable.

Exercise 66:

Let Σ be an alphabet of binary symbols.

- For each ω-tree $t \in \mathbf{T}^{\omega}_{\Sigma}$, we define a language of ω-words $\mathbf{path}(t) \subseteq \Sigma^{\omega}$ as follows:

  $$\mathbf{path}(t) := \{\alpha \in \Sigma^{\omega} \mid \text{ there is path } i_0 i_1 i_2 \cdots \text{ in } t \text{ such that } t(i_0 \cdots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N}\}.$$

  For $B \subseteq \mathbf{T}^{\omega}_{\Sigma}$, let $\mathbf{path}(B) := \bigcup_{t \in B} \mathbf{path}(t)$.

- For a language $L \subseteq \Sigma^{\omega}$ of ω-words, let $\mathbf{tree}(L) := \{t \in \mathbf{T}^{\omega}_{\Sigma} \mid \mathbf{path}(t) \subseteq L\}$.

Prove or refure:

(a) $\mathbf{tree}(\mathbf{path}(B)) = B$

(b) $\mathbf{path}(\mathbf{tree}(L)) = L$

(c) If B is Büchi-recognizable, then $\mathbf{path}(B)$ is also Büchi-recognizable.

(d) If $\mathbf{path}(B)$ is Büchi-recognizable, then B is Büchi-recognizable.

(e) If L is Büchi-recognizable, then $\mathbf{tree}(L)$ is also Büchi-recognizable.