13. Exercises for the Course
‘Automata and Logic’

Exercise 62:

Let $A = (Q, \Sigma, I, \Delta, F)$ be an RL-tree automaton given by:

- $Q = \{1, \ldots , 4\}$,
- $\Sigma = \{g, n, a, b\}$ with $\nu(g) = 2, \nu(n) = 1, \text{ and } \nu(a) = \nu(b) = 0$,
- $I = \{1\}$,
- $\Delta_n(1) = \{1\}, \Delta_n(2) = \{3\}, \Delta_n(3) = \{1, 2\}, \Delta_n(4) = \{1, 3\}$,
- $\Delta_g(1) = \{(1, 1), (1, 2), (3, 4), (4, 1)\}$,
- $\Delta_g(2) = \emptyset$,
- $\Delta_g(3) = Q \times Q$,
- $\Delta_g(4) = \{(1, 2), (1, 4), (2, 4), (2, 2)\}$,
- $F(a) = \{2\}, F(b) = \{2, 3\}$.

Decide whether $L(A) = \emptyset$ or not.

Exercise 63:

Let $\Sigma = \{a, b\}$ be an alphabet with two binary symbols and

$L = \{t \in T_{\Sigma}^\omega | \text{ there is a path in } t \text{ containing only finitely many } a's\}$.

(a) Is $L$ Rabin-recognizable?

(b) Is $L$ Büchi-recognizable?

Exercise 64:

Let $\Sigma$ be an alphabet with arity function containing at least two binary symbols $f$ and $g$. Prove or refute that the $\omega$-tree language $L = \{f(t, t) | t \in T_{\Sigma}^\omega\}$ is Büchi-recognizable.
Exercise 65:

Complete the proof of Theorem 7.11 by showing the following:
Let $\Sigma$ be an alphabet of binary symbols and $L \subseteq T^\omega_\Sigma$ be such that there exists a set $F = \{f_1, \ldots, f_m\}$ of nullary symbols and recognizable tree-languages $L_0, \ldots, L_m \subseteq T_{\Sigma \cup F}$ such that
$$L = L_0 . \left( f_1, \ldots, f_m \right) \left( L_1, \ldots, L_m \right)^\omega . \left( f_1, \ldots, f_m \right).$$
Then $L$ is Büchi-recognizable.

Exercise 66:

Let $\Sigma$ be an alphabet of binary symbols.

- For each $\omega$-tree $t \in T^\omega_\Sigma$, we define a language of $\omega$-words $\text{path}(t) \subseteq \Sigma^\omega$ as follows:
  $$\text{path}(t) := \{ \alpha \in \Sigma^\omega \mid \text{there is path } i_0 i_1 i_2 \cdots \text{ in } t \text{ such that } t(i_0 \cdots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N} \}.$$  
  For $B \subseteq T^\omega_\Sigma$, let $\text{path}(B) := \bigcup_{t \in B} \text{path}(t)$.

- For a language $L \subseteq \Sigma^\omega$ of $\omega$-words, let $\text{tree}(L) := \{ t \in T^\omega_\Sigma \mid \text{path}(t) \subseteq L \}$.

Prove or refute:

(a) $\text{tree}(\text{path}(B)) = B$

(b) $\text{path}(\text{tree}(L)) = L$

(c) If $B$ is Büchi-recognizable, then $\text{path}(B)$ is also Büchi-recognizable.

(d) If $\text{path}(B)$ is Büchi-recognizable, then $B$ is Büchi-recognizable.

(e) If $L$ is Büchi-recognizable, then $\text{tree}(L)$ is also Büchi-recognizable.