Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–39167

4. Exercises for the Course 'Automata and Logic'

Exercise 17:

Let V be an M-variety. Show that $L(V)_\Sigma$ is closed under union **without** using Theorem 1.22 from the lecture.

Exercise 18:

Let Σ be an alphabet. Prove or refute the following claims:

- Every regular language $L \subseteq \Sigma^*$ is accepted by its syntactic monoid.
- If $L \subseteq \Sigma^*$ is accepted by a finite group, then the syntactic monoid of L is a finite group.
- For every regular language $L \subseteq \Sigma^*$, the syntactic monoid M_L is the smallest monoid accepting L; i.e. for every monoid M accepting L, we have $|M_L| \leq |M|$.
- Let $\overleftarrow{w}$ denote the mirror image of the word w; that is, for $w = a_1 \dots a_n$, $\overleftarrow{w} = a_n \dots a_1$. For a language $L \subseteq \Sigma^*$, we define $\overleftarrow{L} := \{\overleftarrow{w} \mid w \in L\}$. **Claim**: if the minimal automaton for L has n states, then the minimal automaton for $\overleftarrow{L}$ has also n states.

Exercise 19:

Let V be the M-variety of all commutative finite groups. Show that there exist a language $L \subseteq \{a\}^*$ such that $L \in L(V)_{\{a\}}$ but $L \notin L(V)_{\{a,b\}}$.

Exercise 20:

Prove or disprove the following: there is a language $L \subseteq \{a, b\}^*$ such that its syntactic semigroup S_L and its syntactic monoid M_L are isomorphic.

Exercise 21:

For each of the following words over the alphabet $\{0, 1\}^k$, give a corresponding interpretation over the predicate symbols $P_1, \dots, P_k$ as discussed in the lecture:
$k = 2$: $(1, 1), (1, 1), (0, 1), (1, 0)$
$k = 3$: $(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)$
$k = 3$: $(1, 1, 0), (1, 0, 1), (1, 1, 1), (1, 1, 0)$
Describe all interpretations that correspond to words of the language $L(((0, 1) \cdot (1, 0))^+) \subseteq (\{0, 1\}^2)^+$.

Exercise 22:

Let $\Sigma = \{a, b\}$. For each of the following regular expressions r_i, give a first-order formula φ_i such that $L(r_i) = L(\varphi_i)$.

(a) $r_1 = \Sigma^*$,

(b) $r_2 = \varepsilon$,

(c) $r_3 = (abb^*)^*$,

(d) $r_4 = a^*b^* + b^*a^*$,

(e) $r_5 = (aaa \cdot \Sigma^*) + b^*$.