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5. Exercises for the Course 'Automata and Logic'

Exercise 23:

Complete the proof of Lemma 2.2 from the lecture by showing that the class $(B_0)_\Sigma$ is closed under union.

Exercise 24:

Let $(S, \circ)$ be a finite semigroup, $m \in S$, and $i, k, l \in \mathbb{N} \setminus \{0\}$ defined as in the proof of Theorem 2.4. Show that, if k is minimal with the property described in the proof, then

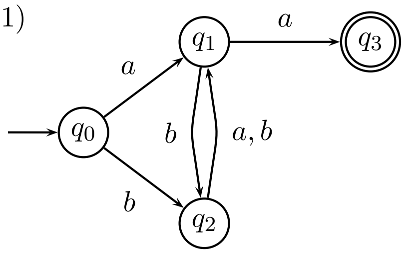
$$(\{m^i, \ldots, m^{i+k-1}\}, \circ, m^l)$$

is a group. Is $(\{m^i, \ldots, m^{i+k-1}\}, \circ, m^l)$ still a group if k is not minimal?

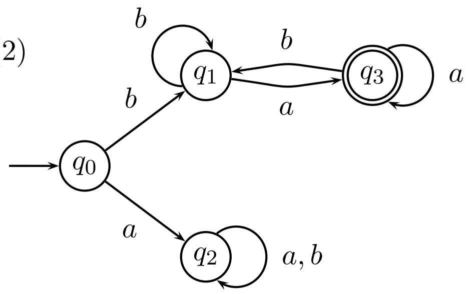
Exercise 25:

Let $\Sigma := \{a, b\}$ and L_1, L_2 be the languages accepted by the automata displayed below. **Use the proof of Corollary 2.10** from the lecture to show that $L_1 \notin (B_0)_\Sigma$ and $L_2 \in (B_0)_\Sigma$. Moreover, represent L_2 as a Boolean combination of languages from the set $\{u\Sigma^* \mid u \in \Sigma^*\} \cup \{\Sigma^* u \mid u \in \Sigma^*\}$.

(1)



(2)



Exercise 26:

Prove or refute the following:

(a) for every alphabet Σ and word $w \in \Sigma^*$, we have $\{w\} \in (B_0)_\Sigma$;

(b) for every two alphabets Σ and Σ' with $\Sigma \subseteq \Sigma'$ and every language $L \subseteq \Sigma^*$, if $L \in (B_0)_\Sigma$, then $L \in (B_0)_{\Sigma'}$;

(c) let $(M, \circ, 1)$ be a monoid and 1 the only idempotent element of M. Then $(M, \circ, 1)$ is a group;

(d) let $(S, \circ)$ be a semigroup and $e \in S$ idempotent. Then $(eSe, \circ, e)$ is the largest submonoid of S with e as unit element;

(e) let $(S, \circ) \in \hat{\mathbb{D}}$. If there exists an element $s \in S$ such that $(S, \circ, s)$ is a monoid, then $|S| = 1$.

Exercise 27:

Let V be the class of all finite semigroups S such that, for all idempotent $e \in S$, we have $Se = e$. Show that V is an S-variety ultimately defined by

$$yx^n = x^n \quad (n \geq 1).$$

Exercise 28:

Let $\Sigma := \{a, b, c, d\}$.

(a) For $L \subseteq \Sigma^*$ with

$$\begin{aligned} L \quad := \quad & \{w \in \Sigma^* \mid w \text{ starts with } a \text{ or } b\} \cap \\ & \{w \in \Sigma^* \mid |w| \geq 3 \text{ and } w \text{ starts and ends with the same symbol}\}, \end{aligned}$$

give a quantifier-free formula φ using the signature $\{Q_a, Q_b, Q_c, Q_d, <, \min, \max, \mathsf{s}, \mathsf{p}\}$ such that $L(\varphi) = L$.

(b) Let

$$\varphi := \neg(\neg Q_a(s(s(p(s(\min))))) \vee (s(\min) < p(p(\max)))).$$

Use the method described in the proof of Proposition 2.11 to describe $L(\varphi)$ as a Boolean combination of languages $\{w\Sigma^* \mid w \in \Sigma^*\} \cup \{\Sigma^* w \mid w \in \Sigma^*\}$.