Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–39167

6. Exercises for the Course 'Automata and Logic'

Exercise 29:

Let Σ be an alphabet. A language $L \subseteq \Sigma^*$ is called *definite* for Σ if there exists an $n \in \mathbb{N}$ such that, for all $w \in L$ it holds that

$$\text{if } w = uv \text{ with } |u| = n, \text{ then } u\Sigma^* \subseteq L.$$

Show that $L \subseteq \Sigma^*$ is definite for Σ if and only if L is a Boolean combination of languages of the form $\{w\Sigma^* \mid w \in \Sigma^*\}$.

Exercise 30:

Let $\Sigma = \{0, 1\}^k$. Show that the following equivalence holds:

$$L \text{ is definite for } \Sigma \iff \begin{array}{l}\text{exists a quantifier-free closed first-order formula } \varphi \text{ over} \\ \text{the signature } \{P_1, \ldots, P_k, <, \min, s\} \text{ with } L(\varphi) = L \setminus \{\varepsilon\}.\end{array}$$

Exercise 31:

Let Σ, Γ be two alphabets and $L \subseteq \Sigma^*$. Prove or refute the following:

(a) $L \in \mathbf{SF}_\Sigma \Longrightarrow L \in \mathbf{S}F_{\Sigma\cup\Gamma}$

(b) $L \in \mathbf{SF}_{\Sigma\cup\Gamma} \Longrightarrow L \in \mathbf{S}F_\Sigma$

Exercise 32:

For $\Sigma := \{a, b\}$, check whether the following languages are star-free:

(a) $L := (ab)^*$

(b) $L := \{w \mid |w|_a = 3k \text{ for some } k \in \mathbb{N}\}$

(c) $L := a(aba)^*b$.

Use Theorem 3.6 from the lecture or give a star-free description of the language.