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7. Exercises for the Course 'Automata and Logic'

Exercise 33:

Let $\Sigma = \{a\}$. Recall from the lecture that $L_{k,n}$ denotes the set of all first-order formulas (over the non-logical symbols $=, <$, and Q_a) containing k free variables and having quantifier depth at most n. For the following combinations of k, n, determine a *finite* set $\Gamma_{k,n}$ such that, for every formula $\varphi \in L_{k,n}$ there is a $\psi \in \Gamma_{k,n}$ with $\varphi \equiv \psi$.

(a) $k = 1, n = 0$;

(b) $k = 2, n = 0$;

(c) $k = 0, n = 1$;

(d) $k = 1, n = 1$.

Exercise 34:

For the combinations k, n of the previous exercise, determine the equivalence classes of $\equiv_{k,n}$.

Exercise 35:

Give the formulas φ_W for each equivalence class W of $\equiv_{2,0}$. Then, determine a finite disjunction of formulas φ_W for $\equiv_{2,0}$-classes which is equivalent to the formulas:

(a) true

(b) $\neg(x < y) \vee x = y$

(c) false

Exercise 36:

Consider the Ehrenfeucht-Fraïssé games on the words

(a) ab and ba,

(b) $aaabaaa$ and $aabaaa$.

Determine the $k \in \{1, 2, 3, 4\}$ such that Player I has a winning strategy in k moves.

Exercise 37:

Consider the Ehrenfeucht-Fraïssé game on words a^i and a^j with $i < j$.

(a) Describe and optimal winning strategy for Player I; that is, a strategy such that Player I wins with a minimal number of moves.

(b) Prove that Player I has a winning strategy on a^i and a^j (with $i < j$) in m moves if $i < 2^m - 1$.