Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–39167

8. Exercises for the Course 'Automata and Logic'

Exercise 38:

Let $\Sigma := \{a, b\}$ and $L \subseteq \Sigma^*$ be defined by the regular expression $(a^*bb^*)^*$. Show that

$$\lim L = \{\alpha \in \Sigma^\omega \mid \text{if } \alpha(i,i) = a, \text{ then there is a } j > i \text{ with } \alpha(j,j) = b\}.$$

Exercise 39:

Give Büchi automata that recognize the following ω-regular languages over the alphabet $\Sigma := \{a, b, c\}$:

(a) $\{\alpha \in \Sigma^\omega \mid \exists i \in \mathbb{N} : \alpha(i, i+2) = abc\}$

(b) $\{\alpha \in \Sigma^\omega \mid \{i \in \mathbb{N} \mid \alpha(i, i+2) = abc\} \text{ is infinite}\}$

(c) $(a^+b^+c^+)^\omega$

Exercise 40:

(a) Show that the construction used in the proof of Lemma 4.7.1 does not work for automata whose initial state is reachable from another state.

(b) Complete the proof of Lemma 4.7 from the lecture by showing that

$$\text{if } L_1, L_2 \subseteq \Sigma^\omega \text{ are Büchi recognizable, then } L_1 \cup L_2 \text{ is Büchi recognizable}$$

Exercise 41:

Let Σ be an alphabet and $L, L_1, L_2 \subseteq \Sigma^*$. Prove or refute:

(a)
- $(L_1 \cup L_2)^\omega \subseteq L_1^\omega \cup L_2^\omega$
- $(L_1 \cup L_2)^\omega \supseteq L_1^\omega \cup L_2^\omega$

(b)
- $\lim(L_1 \cup L_2) \subseteq \lim L_1 \cup \lim L_2$
- $\lim(L_1 \cup L_2) \supseteq \lim L_1 \cup \lim L_2$

(c)
- $L^\omega \subseteq \lim L^+$
- $L^\omega \supseteq \lim L^+$

(d)
- $\lim(L_1 \cdot L_2) \subseteq L_1 \cdot L_2^\omega$
- $\lim(L_1 \cdot L_2) \supseteq L_1 \cdot L_2^\omega$