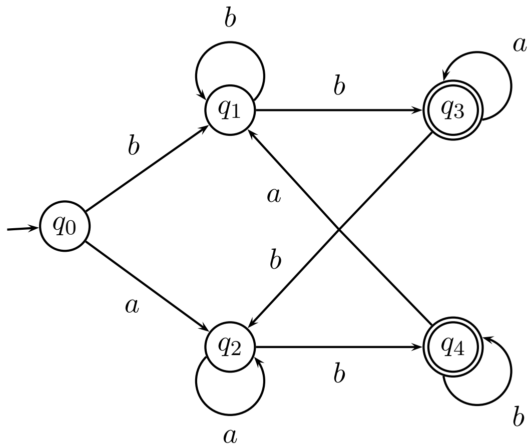
Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–39167

9. Exercises for the Course 'Automata and Logic'

Exercise 42:

Let $\Sigma := \{a, b\}$ and $L \subseteq \Sigma^\omega$ the language recognized by the following Büchi automaton:



Find a number $n \geq 1$ and regular languages $U_1, V_1, \ldots, U_n, V_n \subseteq \Sigma^*$ such that

$$\bigcup_{i=1}^{n} U_i \cdot V_i^\omega = L.$$

Exercise 43:

Let Σ be an alphabet. Prove the following:

(a) If $L \subseteq \Sigma^+$ is regular, then there exists a finite non-deterministic automaton $\mathcal{A}$ with only **one** final state such that $L = L(\mathcal{A})$.

(b) If $L \subseteq \Sigma^*$ is regular, then there exists a finite non-deterministic automaton $\mathcal{A}$ with at most **two** final state such that $L = L(\mathcal{A})$.

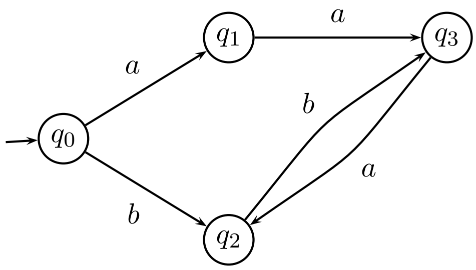
(c) There is **no** $k \geq 1$ such that the following holds:

> If $L \subseteq \Sigma^\omega$ is Büchi recognizable, then there exists a Büchi automaton $\mathcal{A}$ with at most k final states such that $L = L_\omega(\mathcal{A})$.

Hint: consider the languages $a^\omega \cup b^\omega, a^\omega \cup b^\omega \cup c^\omega, \ldots$

Exercise 44:

Consider Büchi automata using the following transition system:



Check whether the recognized language is empty for the following sets of final states:

(a) $F = \{q_0, q_1\}$

(b) $F = \{q_2, q_3\}$

(c) $F = \{q_1, q_3\}$

Exercise 45:

For a finite automataon $\mathcal{A}$, let $\mathcal{A}_{\mathsf{det}}$ denote the minimal deterministic automaton such that $L(\mathcal{A}) = L(\mathcal{A}_{\mathsf{det}})$. Prove or refute the following:

(a) $\lim L(\mathcal{A}) = L_\omega(\mathcal{A}_{\mathsf{det}})$

(b) $L_\omega(\mathcal{A}) \subseteq L_\omega(\mathcal{A}_{\mathsf{det}})$

(c) $L_\omega(\mathcal{A}_{\mathsf{det}}) \subseteq L_\omega(\mathcal{A})$

Exercise 46:

Let $(r_n)_{n\geq 0}$ be a sequence of real numbers. Show that there exists an infinite subsequence of $(r_n)_{n\geq 0}$ that is

- strictly increasing, or
- strictly decreasing, or
- constant.