TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Fuzzy Description Logics

Solutions to Exercise Sheet 1

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Exercise 4

We check whether the subsumption relation

$$A \sqsubseteq Y$$

holds with respect to the normalized TBox $\mathcal{T}'$ that contains the following axioms.[1]

$$\begin{aligned} A &\sqsubseteq \exists r.X_1 \\ X_1 &\sqsubseteq C \\ X_1 &\sqsubseteq D \\ \exists r.B &\sqsubseteq X_3 \\ B \sqcap X_3 &\sqsubseteq X_2 \\ X_4 &\sqsubseteq \exists r.B \\ X_2 &\sqsubseteq \exists r.X_4 \\ \exists r.A &\sqsubseteq X_5 \\ \exists r.X_5 &\sqsubseteq B \\ C &\sqsubseteq B \\ C &\sqsubseteq \exists r.A \end{aligned}$$

and the two axioms

$$\begin{aligned} \exists r.X_6 &\sqsubseteq Y \\ \exists r.B &\sqsubseteq X_6. \end{aligned}$$

The application of completion rules yields

- (R2), $A \sqsubseteq \exists r.X_1$, $A \in S(A) \rightsquigarrow$ add r to $R(A, X_1)$
- (R1), $X_1 \sqsubseteq C$, $X_1 \in S(X_1) \rightsquigarrow$ add C to $S(X_1)$
- (R2), $C \sqsubseteq \exists r.A$, $C \in S(X_1) \rightsquigarrow$ add r to $R(X_1, A)$
- (R3), $\exists r.A \sqsubseteq X_5$, $A \in S(A)$, $r \in R(X_1, A) \rightsquigarrow$ add X_5 to $S(X_1)$

[1] Indices may be different from the tutorial.

- (R3), $\exists r.X_5 \sqsubseteq B$, $X_5 \in S(X_1)$, $r \in R(A, X_1) \rightsquigarrow$ add B to $S(A)$
- (R1), $C \sqsubseteq B$, $C \in S(X_1) \rightsquigarrow$ add B to $S(X_1)$
- (R3), $\exists r.B \sqsubseteq X_3$, $B \in S(X_1)$, $r \in R(A, X_1) \rightsquigarrow$ add X_3 to $S(A)$
- (R1), $B \sqcap X_3 \sqsubseteq X_2$, $B \in S(A)$, $X_3 \in S(A) \rightsquigarrow$ add X_2 to $S(A)$
- (R2), $X_2 \sqsubseteq \exists r.X_4$, $X_2 \in S(A) \rightsquigarrow$ add r to $R(A, X_4)$
- (R2), $X_4 \sqsubseteq \exists r.B$, $X_4 \in S(X_4) \rightsquigarrow$ add r to $R(X_4, B)$
- (R3), $\exists r.B \sqsubseteq X_6$, $B \in S(B)$, $r \in R(X_4, B) \rightsquigarrow$ add X_6 to $S(X_4)$
- (R3), $\exists r.X_6 \sqsubseteq Y$, $X_6 \in S(X_4)$, $r \in R(A, X_4) \rightsquigarrow$ add Y to $S(A)$

Completion yields $Y \in S(A)$. Therefore $A \sqsubseteq Y$ holds with respect to $\mathcal{T}' \cup \{\exists r.X_6 \sqsubseteq Y, \exists r.B \sqsubseteq X_6\}$ and $A \sqsubseteq \exists r.\exists r.B$ holds with respect to $\mathcal{T}'$ and $\mathcal{T}$.