Fuzzy Description Logics

Exercise Sheet 1

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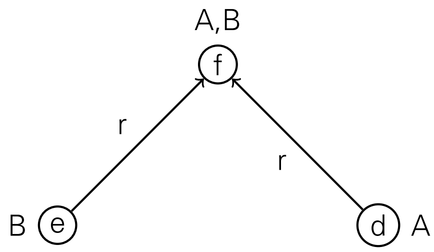
Exercise 1

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta_{\mathcal{I}} = \{d, e, f\}$.



List all the elements of $C^{\mathcal{I}}$ for each of the following concept descriptions C.

a) $\mathcal{EL}$:
   - $\exists r.(A \sqcap B)$,
   - $B \sqcap \exists r.A$, and

b) $\mathcal{ALC}$:
   - $A \sqcup B$,
   - $\neg A \sqcup \neg B$,
   - $\forall r.(B \sqcap A)$

Exercise 2

Consider the ABox

$$\mathcal{A} = \{ A(d), B(e), A(f), B(f), \\ r(e, f), r(d, f)\}.$$

a) Present a graphical representation of the ABox $\mathcal{A}$.

b) For each of the concept descriptions C from Exercise 1 list all individuals that are instances of C with respect to $\mathcal{A}$.

Exercise 3

Consider the TBox $\mathcal{T}$ having the following axioms:

$$\begin{aligned} A &\sqsubseteq \exists r.(C \sqcap D), \\ B \sqcap \exists r.B &\sqsubseteq \exists r.\exists r.B, \\ \exists r.\exists r.A &\sqsubseteq B, \\ C &\sqsubseteq B \sqcap \exists r.A\}. \end{aligned}$$

Normalize $\mathcal{T}$ using the normalization rules

(NF1) $C \sqcap \hat{D} \sqsubseteq E \;\rightsquigarrow\; \hat{D} \sqsubseteq A, C \sqcap A \sqsubseteq E$,

(NF2) $\exists r.\hat{D} \sqsubseteq E \;\rightsquigarrow\; \hat{D} \sqsubseteq A, \exists r.A \sqsubseteq E$,

(NF3) $B \sqsubseteq \exists r.\hat{C} \;\rightsquigarrow\; A \sqsubseteq \hat{C}, B \sqsubseteq \exists r.A$,

(NF4) $\hat{C} \sqsubseteq \hat{D} \;\rightsquigarrow\; \hat{C} \sqsubseteq A, A \sqsubseteq \hat{D}$, and

(NF5) $C \sqsubseteq D \sqcap E \;\rightsquigarrow\; C \sqsubseteq D, C \sqsubseteq E$

where $\hat{C}, \hat{D} \notin \mathcal{N}_C \cup \{\top\}$ and A is a new concept name.

Exercise 4

Verify whether the subsumption relation

$$A \sqsubseteq \exists r.\exists r.B$$

holds with respect to the TBox $\mathcal{T}$ from Exercise 3 using the completion rules

(R1) $A_1 \sqcap A_2 \sqsubseteq B \in \mathcal{T}, A_1, A_2 \in S(A) \;\rightsquigarrow$ add B to $S(A)$,

(R2) $A_1 \sqsubseteq \exists r.B \in \mathcal{T}, A_1 \in S(A) \;\rightsquigarrow$ add r to $R(A, B)$, and

(R3) $\exists r.A_1 \sqsubseteq B \in \mathcal{T}, A_1 \in S(A_2), r \in R(A, A_2) \;\rightsquigarrow$ add B to $S(A)$,

where each concept name A is initially labelled with $S(A) = \{A, \top\}$ and each pair (A, B) is initially labelled with $R(A, B) = \emptyset$.

Exercise 5

Use a tableau algorithm to decide whether the following subsumption holds:

$$\neg\forall r.A \sqcap \forall r.C \sqsubseteq_{\mathcal{T}} \forall r.E$$

where $\mathcal{T} = \{C \equiv (\exists r.\neg B) \sqcap \neg A, \quad D \equiv \exists r.B, \quad E \equiv \neg(\exists r.A) \sqcap \exists r.D\}$.