TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Fuzzy Description Logics

Exercise Sheet 4

Dr. Rafael Peñaloza / Dr. Felix Distel
Winter Semester 2011/2012

Exercise 1

Show that the completion algorithm for Gödel-$\mathcal{EL}$ terminates in polynomial time.

Exercise 2

In crisp $\mathcal{ALCH}$ (i.e. $\mathcal{ALC}$ extended with role inclusions) express that

a) Someone who has a sidekick also has a teammate.

b) A person's sidekick is always that person's teammate.

Exercise 3

Let $\mathcal{T}$ be the TBox $\mathcal{T} = \{\langle \exists r.A \sqsubseteq A, 0.5\rangle\}$. Using the translation to crisp $\mathcal{ALCH}$ from the lecture check whether $A \sqcap \forall r.\bot$ is subsumed by $\exists r.(A \sqcup B)$ wrt $\mathcal{T}$ to degree 0.5 when the semantics are restricted to witnessed models.

Exercise 4

Prove or disprove:

a) There is an interpretation $\mathcal{I}$ and $x \in \Delta^{\mathcal{I}}$ such that $(\forall r.\neg\neg A)^{\mathcal{I}}(x) \Rightarrow (\forall r.A)^{\mathcal{I}}(x) = 0$.

b) There is a witnessed interpretation $\mathcal{I}$ and $x \in \Delta^{\mathcal{I}}$ such that $(\forall r.\neg\neg A)^{\mathcal{I}}(x) \Rightarrow (\forall r.A)^{\mathcal{I}}(x) = 0$.

Exercise 5

As an alternative to the Gödel-negation one can define the *involutive negation* $\sim$ whose semantics is defined as

$$(\sim C)^{\mathcal{I}}(x) = 1 - C^{\mathcal{I}}(x)$$

for all interpretations $\mathcal{I}$ and all $x \in \Delta^{\mathcal{I}}$. Using Gödel-$\mathcal{EL}$ and involutive negation construct a TBox $\mathcal{T}$ and an ABox $\mathcal{A}$ with the following properties.

a) There is a constant $q \in (0, 1)$ and a concept name A occuring in $\mathcal{T}$ such that $A^{\mathcal{I}}(x) = q$ for all models $\mathcal{I}$ of $\mathcal{T}$ and all $x \in \Delta^{\mathcal{I}}$.

b) $\mathcal{A}$ is inconsistent but becomes consistent if all occurrences of $\neg$ are replaced by $\sim$.