TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute for Theoretical Computer Science, Chair for Automata Theory

Fuzzy Description Logics

Exercise Sheet 5

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Exercise 1

Show that the following three binary operators are continuous t-norms:

a) *Łukasiewicz t-norm:* $x \otimes y = \max\{x + y - 1, 0\}$,

b) *Product t-norm:* $x \otimes y = x \cdot y$,

c) *Gödel t-norm:* $x \otimes y = \min\{x, y\}$.

Exercise 2

Show that for every continuous t-norm $\otimes$ and its residuum $\Rightarrow$, and every $x, y, z \in [0, 1]$

a) $x \leq y$ iff $(x \Rightarrow y) = 1$,

b) $(1 \Rightarrow x) = x$,

c) $x \Rightarrow (y \Rightarrow z) = (x \otimes y) \Rightarrow z$.

Exercise 3

A partial order on the set of all t-norms can be defined naturally as follows. Let $\otimes_1$ and $\otimes_2$ denote two t-norms. We write

$$\otimes_1 \leq \otimes_2 :\Leftrightarrow \forall u, v \in [0, 1] : u \otimes_1 v \leq u \otimes_2 v.$$

Find two t-norms $\otimes_{\min}$ and $\otimes_{\max}$ such that every t-norm $\otimes$ satisfies $\otimes_{\min} \leq \otimes \leq \otimes_{\max}$.

Exercise 4

Check for which of the three t-norms from Exercise 1 the following equalities hold. Provide a proof or a counterexample when appropriate.

a) $\ominus \ominus x = x$

b) $x \Rightarrow y = \ominus x \oplus y$

c) $x \oplus y = \ominus(\ominus x \otimes \ominus y)$

d) $x \otimes \ominus x = 1$

e) $x \otimes (x \Rightarrow y) = x \otimes y$

Exercise 5

An element $x \in [0, 1]$ is called *idempotent* for a t-norm $\otimes$ if it satisfies $x \otimes x = x$. Using ordinal sums, construct a continuous t-norm where exactly the values 0, 1 and the values from the interval $[0.4, 0.6]$ are idempotent.