Fuzzy Description Logics

Exercise Sheet 7

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Exercise 1

We consider the logic $\otimes - \mathcal{ALC}$ where $\otimes$ is

a) the Lukasiewicz-t-norm, or

b) the Product-t-norm.

Give an ontology $\mathcal{O}$ such that

- $\mathcal{O}$ is consistent, and
- in any model of $\mathcal{O}$ infinitely many truth values occur.

Exercise 2

Let $\mathbf{L} = (L, \wedge, \vee)$ be a lattice. Prove that the relation $\leq$ as defined in the lecture has the following properties.

a) $\leq$ is a partial order, i.e. it is associative, commutative and anti-symmetric,

b) $a \wedge b$ is the infimum of a and b with respect to $\leq$, and

c) $a \vee b$ is the supremum of a and b with respect to $\leq$.

Exercise 3

Show that the following structures are lattices. For each of them draw a Hasse-diagram of the order relation. In each case find an operation $\sim$ that turns it into a De Morgan lattice.

a) $(\mathcal{P}(\{1, 2, 3\}), \cap, \cup)$, and

b) the three valued logic $(\{1, 0, ?\}, \wedge, \vee)$ where we define

$$x \wedge y = \begin{cases} 1 & x = y = 1 \\ 0 & x = 0 \text{ or } y = 0 \\ ? & \text{otherwise} \end{cases}, \qquad \text{and } x \vee y = \begin{cases} 0 & x = y = 0 \\ 1 & x = 1 \text{ or } y = 1 \\ ? & \text{otherwise} \end{cases}.$$

c) (D_{140}, gcd, lcm) where D_{140} is the set of all natural numbers that divide 140, and gcd and lcm denote the greatest common divisor and the least common multiple.

Exercise 4

Let $(L, \wedge, \vee)$ be a distributive lattice. Define $x \otimes y = x \wedge y$ for all $x, y \in L$, and $x \Rightarrow y = \bigvee\{z \mid x \wedge z \leq y\}$ for all $x, y \in L$. Prove that $(L, \wedge, \vee, \otimes, \Rightarrow)$ is a residuated lattice.

Is this also true when L is not distributive?

Exercise 5

Let the looping tree automaton $\mathcal{A} = (Q, I, \Delta)$ over binary trees be given by

- $Q = \{q_0, q_1, q_2\}$,
- $I = \{q_0\}$, and
- $\Delta = \{(q_0, q_0, q_1), (q_0, q_0, q_2), (q_1, q_1, q_2)\}$.

Does $\mathcal{A}$ have a run?