

Introduction to Automatic Structures

Exercise Sheet 1

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Exercise 1

Let $\Sigma = \{a, b\}$. Let L_1 be the language defined by

$$L_1 = \{w \in \Sigma^* \mid \text{the number of occurrences of } a \text{ in } w \text{ is odd}\}$$

- Prove that L_1 is regular by giving a regular expression for it.
- Construct a finite automaton M such that $L_1 = L(M)$.

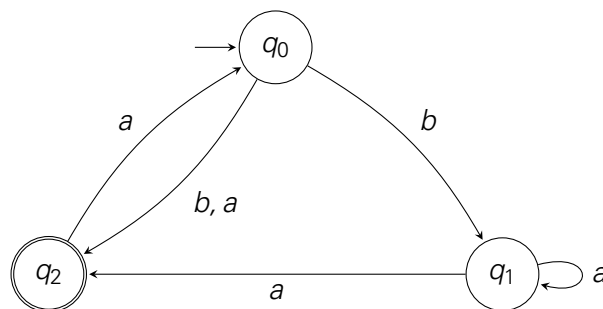
Exercise 2

Show that regular languages are closed under

- union,
- intersection,
- concatenation, and
- Kleene star.

Exercise 3

Let the non-deterministic finite automaton $M := (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_1, q_2\})$ be given by the following transition system.



- Apply the power set construction to M in order to obtain a *deterministic* finite automaton that accepts the same language as M .
- Use your result to construct a finite automaton \overline{M} that accepts the complement of this language.

Exercise 4

Consider the alphabet $\Sigma = \{0, 1\}$. We assume that in its initial configuration a natural number $n \in \mathbb{N}$ is written on the tape of a Turing Machine in *binary encoding*. Construct

- a) a Turing Machine TM_1 that computes $n + 1$, and
- b) a Turing Machine TM_2 that computes $2n$.