Exercise 1

Let $\Sigma = \{a, b\}$. Let $L_1$ be the language defined by

$$L_1 = \{w \in \Sigma^* \mid \text{the number of occurrences of } a \text{ in } w \text{ is odd}\}$$

a) Prove that $L_1$ is regular by giving a regular expression for it.

b) Construct a finite automaton $M$ such that $L_1 = L(M)$.

Exercise 2

Show that regular languages are closed under

a) union,

b) intersection,

c) concatenation, and

d) Kleene star.

Exercise 3

Let the non-deterministic finite automaton $M := (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_0, q_2\})$ be given by the following transition system.

```
a
\rightarrow\quad q_0
\quad \quad a
\quad b
\downarrow
q_1
\quad a
q_2
\downarrow
\quad b, a
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a) Apply the power set construction to $M$ in order to obtain a deterministic finite automaton that accepts the same language as $M$.

b) Use your result to construct a finite automaton $\overline{M}$ that accepts the complement of this language.
Exercise 4
Consider the alphabet $\Sigma = \{0, 1\}$. We assume that in its initial configuration a natural number $n \in \mathbb{N}$ is written on the tape of a Turing Machine in *binary encoding*. Construct

a) a Turing Machine $TM_1$ that computes $n + 1$, and

b) a Turing Machine $TM_2$ that computes $2n$. 