

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Automatic Structures

Exercise Sheet 2

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Notice

This is Part 1 of this exercise sheet. A second part will be available on Monday.

Exercise 5

Which of the following structures are automatic structures?

a) $(\{a, b\}^*, R_1)$ where

$$R_1 := \{(u, v) \mid u, v \in \{a, b\}^*, \text{ for all } i : u(i) = a \text{ implies } v(i) = a\}.$$

b) $(\{a\}^*, R_2, R_3)$ where

 $R_2 := \{(u, v, w) \mid u, v, w \in \{a\}^*, \text{length}(u) > \text{length}(v) > \text{length}(w)\}.$

and

 $R_3 := \{(u, v, w) \mid u, v, w \in \{a\}^*, \text{length}(u) > \text{length}(v) + \text{length}(w)\}.$

c) The configuration space (C, R) of a Turing machine *TM*. Here *C* consists of words of the form w_0qw_1 , representing the configuration (q, w, p) with $w = w_0w_1$ and $|w_0| = p$. Two configurations w_0qw_1 and u_0ru_1 are in relation *R* if there is a one-step transition from w_0qw_1 to u_0ru_1 .

Exercise 6

Let *L* be a regular language. Let *R* be an automatic relation. Show that universal projections of R with respect to *L* are automatic relations.