TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Introduction to Automatic Structures

## Exercise Sheet 3

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### Exercise 9

Do the following structures have automatic presentations?

a) $(\mathbb{Z}, +)$

b) the structure from Exercise 5 b), i.e. $(\{a\}^*, R_2, R_3)$ where

$$R_2 := \{(u, v, w) \mid u, v, w \in \{a\}^*, \text{length}(u) > \text{length}(v) > \text{length}(w)\}$$

and

$$R_3 := \{(u, v, w) \mid u, v, w \in \{a\}^*, \text{length}(u) > \text{length}(v) + \text{length}(w)\}$$

### Exercise 10

Let $\Sigma$ be a finite alphabet, $R$ a binary relation on $\Sigma^*$ such that $(\Sigma^*, R)$ is automatic. Let $f$ be the function that maps every word $w \in \Sigma^*$ to $f(w) = \overleftarrow{w}$, i.e. the word $w$ written backwards. Define $R'$ to be the relation

$$R' = \{(f(w_1), f(w_2)) \mid (w_1, w_2) \in R\}.$$

Prove or disprove the following statements.

a) The structure $(\Sigma^*, R')$ is always automatic.

b) The structure $(\Sigma^*, R')$ always has an automatic presentation.

### Exercise 11

Let $\mathcal{A} = (A, P)$ be a structure where $P \subseteq A$ is a unary relation and $A$ is countably infinite. Show that $\mathcal{A}$ has an automatic presentation.

### Exercise 12

**(Constant Growth Lemma)**

Let $f : \{0, 1\}^* \times \{0, 1\}^* \to \{0, 1\}^*$ be a bijection. Use the Constant Growth Lemma to prove that $\mathcal{A} = (\{0, 1\}^*, R)$ with

$$R = \{(u, v, f(u, v)) \mid u, v \in \{0, 1\}^*\}$$

is not automatic.