TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Automatic Structures

Exercise Sheet 4

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Exercise 13

Let Σ be a totally ordered alphabet and $\leq_{\text{lex}}$ the lexicographic order on Σ^*. The length-lexicographic order $\leq_{\text{llex}}$ is the binary relation on Σ^* where

$$w_1 \leq_{\text{llex}} w_2 \text{ iff } \text{length}(w_1) < \text{length}(w_2) \text{ or } \text{length}(w_1) = \text{length}(w_2) \text{ and } w_1 \leq_{\text{lex}} w_2.$$

Show that $(\Sigma^*, \leq_{\text{llex}})$ is automatic.

Exercise 14

Complete the construction of the automaton M_α in the proof of Theorem 3.1 for the remaining cases: equality, negation, disjunction.

Exercise 15

Let $\mathcal{A}$ be a structure. Which of the following statements are true? Justify your answer.

a) If model checking is decidable for $\mathcal{A}$ then the FOL theory of $\mathcal{A}$ is also decidable.

b) If there is an algorithm for query containment for $\mathcal{A}$ then there is also an algorithm that decides model checking for $\mathcal{A}$.

c) If the FOL theory of $\mathcal{A}$ is decidable then model checking for $\mathcal{A}$ is also decidable.

Exercise 16

Consider the structure

$$\mathcal{A} = (\{0, 1\}^*; \preceq, S_0, S_1, \mathit{EqualLength})$$

from Example 2.5, where $\preceq$ is the prefix relation, S_0 and S_1 append 0 and 1, respectively, and *EqualLength* checks for equal length. For each of the following relations R_i give a FOL-formula ϕ_i such that $(\mathcal{A}, \bar{a}) \models \phi_i(\bar{x})$ iff $\bar{a} \in R_i$.

a) $R_1 = \{(u, v) \mid u, v \in \{0, 1\}^*, \text{length}(u) \leq \text{length}(v)\}$,

b) $R_2 = \{(u, v) \mid \text{the } |v|\text{-th symbol in } u \text{ is } 0\}$, and

c) $R_3 = \{(u, v, w) \mid u \text{ and } v \text{ differ in the } |w|\text{-th symbol}\}$.

Exercise 17

Let $(L, \leq)$ be a poset.

a) Give a FOX-formula to describe the pairs (x, y) such that the interval $[x, y]$ contains an even number of elements.

b) Give a FO∞-formula that characterizes trees with infinite outdegree.